

8.3 The Double-Angle and Half-Angle Formulas

OBJECTIVE 1: Understanding the Double-Angle Formulas

The Double Angle Formulas

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

OBJECTIVE 3: Understanding the Half-Angle Formulas

The Half-Angle Formulas

$$\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos \alpha}{2}} \quad \text{for } \frac{\alpha}{2} \text{ in Quadrant I or Quadrant II}$$

$$\sin\left(\frac{\alpha}{2}\right) = -\sqrt{\frac{1 - \cos \alpha}{2}} \quad \text{for } \frac{\alpha}{2} \text{ in Quadrant III or Quadrant IV}$$

$$\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 + \cos \alpha}{2}} \quad \text{for } \frac{\alpha}{2} \text{ in Quadrant I or Quadrant IV}$$

$$\cos\left(\frac{\alpha}{2}\right) = -\sqrt{\frac{1 + \cos \alpha}{2}} \quad \text{for } \frac{\alpha}{2} \text{ in Quadrant II or Quadrant III}$$

$$\tan\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} \quad \text{for } \frac{\alpha}{2} \text{ in Quadrant I or Quadrant III}$$

$$\tan\left(\frac{\alpha}{2}\right) = -\sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} \quad \text{for } \frac{\alpha}{2} \text{ in Quadrant II or Quadrant IV}$$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1 - \cos \alpha}{\sin \alpha} \quad \text{for } \frac{\alpha}{2} \text{ in any quadrant}$$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{\sin \alpha}{1 + \cos \alpha} \quad \text{for } \frac{\alpha}{2} \text{ in any quadrant}$$

OBJECTIVE 4: Using the Double-Angle and Half-Angle Formulas to Verify Identities

OBJECTIVE 5: Using the Double-Angle and Half-Angle Formulas to Evaluate Expressions Involving Inverse Trigonometric Functions