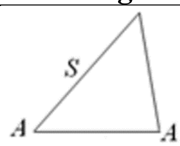
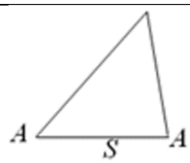
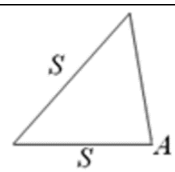
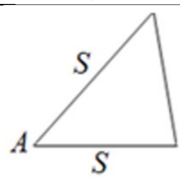
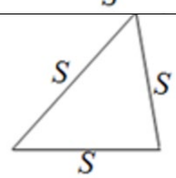
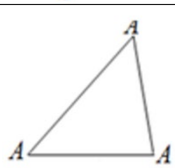
9.2 The Law of Sines

OBJECTIVE 1: Determining if the Law of Sines Can Be Used to Solve an Oblique Triangle

The Law of Sines: If *A*, *B*, and *C* are the measures of the angles of any triangle and if *a*, *b*, and *c* are the lengths of the sides opposite the corresponding angles, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

Triangle	Description of Case	Abbreviation of Case
	Side-Angle-Angle: two angles and a side opposite one of the angles are known	SAA
	Angle-Side-Angle: two angles and the side between them are known	ASA
	Side-Side-Angle: two sides and an angle opposite one of the sides are known	SSA
	Side-Angle-Side: two sides and the angle between them are known	SAS
	Side-Side-Side: all three sides are known	SSS
	Angle-Angle-Angle: all three angles are known	AAA

Since the Law of Sines uses proportions that involve both angles and sides, the following pieces of information are needed in order to solve an oblique triangle using the Law of Sines:

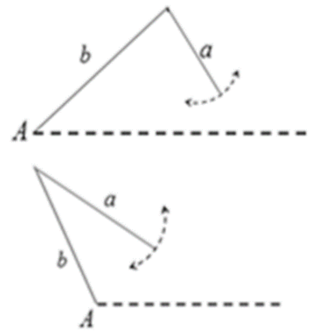
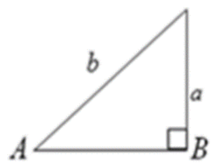
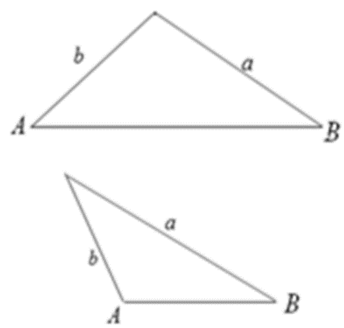
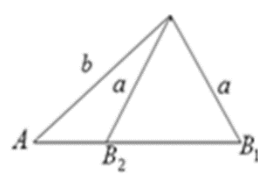
1. The measure of an angle must be known
2. The length of the side opposite the known angle must be known
3. At least one more side or one more angle must be known

The first three cases listed in the table above involve situations where this information is known. Therefore, the Law of Sines can be used to solve the SAA, ASA, and SSA cases.

OBJECTIVE 2: Using the Law of Sines to Solve the SAA Case or ASA Case

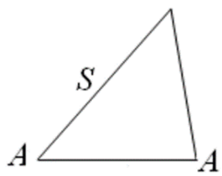
When the measure of any two angles of an oblique triangle is known and the length of any side is known, always start by determining the measure of the unknown angle. Then use appropriate Law of Sines proportions to solve for the lengths of the remaining unknown sides. Whenever possible, we will avoid using rounded information to solve for the remaining parts of the triangle. When this cannot be avoided, we will agree to use information rounded to one decimal place unless some other guideline is stated.

OBJECTIVE 3: Using the Law of Sines to Solve the SSA Case

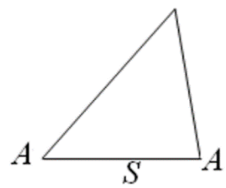
Value of sin *B*	**Number of Triangles**	**Possible Triangles**	**Description**
$\sin B > 1$	No triangle		No angle B exists and side a is too short to reach the opposite side.
$\sin B = 1$	One right triangle		The measure of B is 90°.
$0 < \sin B < 1$	One oblique triangle		If there is one solution for B, then the triangle is oblique with either 3 acute angles or one obtuse angle.
$0 < \sin B < 1$	Two oblique triangles		If there are two solutions for B (B_1 and B_2), then there are two oblique triangles: one with 3 acute angles and one with one obtuse angle.

Here is a summary of the three cases for which the Law of Sines can be used.

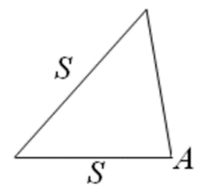
If *S* represents a given side of a triangle and if *A* represents a given angle of a triangle, then the Law of Sines can be used to solve the three oblique triangle cases SAA, ASA, and SSA (Ambiguous Case). Each case is illustrated below.



The SAA Case



The ASA Case



The SSA Case
(The Ambiguous Case)

OBJECTIVE 4: Using the Law of Sines to Solve Applied Problems Involving Oblique Triangles