9.3 The Law of Cosines

OBJECTIVE 1: Determining if the Law of Sines or the Law of Cosines Should Be Used to Begin to Solve an Oblique Triangle

Recall from Section 9.2 that the Law of Sines can be used to solve the SAA Case, the ASA Case, and the SSA (Ambiguous) Case. The AAA Case never defines a unique triangle, so we will ignore it. The remaining two cases, the SAS Case and the SSS Case, can be solved using the Law of Cosines.

The Law of Cosines: If *A*, *B*, and *C* are the measures of the angles of any triangle and if *a*, *b*, and *c* are the lengths of the sides opposite the corresponding angles, then

$$a^2 = b^2 + c^2 - 2bc\cos A,$$
$$b^2 = a^2 + c^2 - 2ac\cos B, \text{ and}$$
$$c^2 = a^2 + b^2 - 2ab\cos C.$$

The Alternate Form of the Law of Cosines: If *A*, *B*, and *C* are the measures of the angles of any triangle and if *a*, *b*, and *c* are the lengths of the sides opposite the corresponding angles, then

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc},$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}, \text{ and}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

OBJECTIVE 2: Using the Law of Cosines to Solve the SAS Case

Solving a SAS Oblique Triangle

1. Use the Law of Cosines to determine the length of the missing side.
2. Determine the measure of the smaller of the remaining two angles using the Law of Sines or using the alternate form of the Law of Cosines. (This angle will always be acute.)
3. Use the fact that the sum of the measures of the three angles of a triangle is 180° to determine the measure of the remaining angle.

Whenever possible, we will avoid using rounded information to solve for the remaining parts of the triangle. When this cannot be avoided, we will agree to use information rounded to one decimal place unless some other guideline is stated.

OBJECTIVE 3: Using the Law of Cosines to Solve the SSS Case

Solving a SSS Oblique Triangle

1. Use the alternate form of the Law of Cosines to determine the measure of the largest angle. This is the angle opposite the longest side.
2. Determine the measure of one of the remaining two angles using the Law of Sines or the alternate form of the Law of Cosines. (This angle will always be acute.)
3. Use the fact that the sum of the measures of the three angles of a triangle is 180° to determine the measure of the remaining angle**.**

OBJECTIVE 4: Using the Law of Cosines to Solve Applied Problems Involving Oblique Triangles