Section 2.4 Parallel and Perpendicular Lines

Given any two *distinct* lines in the Cartesian plane, the two lines will either intersect or they will not. In this section, we will investigate the nature of two lines that do not intersect (**parallel lines**) and then discuss the special case of two lines that intersect at a right angle (**perpendicular lines**). These two cases are interesting because we need only know the slope of the two lines to determine whether or not the lines are parallel, perpendicular or neither.

Objective 1: Understanding the Definition of Parallel Lines

Two lines are parallel if they do not intersect, or in other words, if the lines do not share any common points. Since parallel lines do not intersect, the ratio of the vertical change (rise) to the horizontal change (run) of each line must be equivalent. In other words, parallel lines have the same slope.

Theorem: Two distinct non-vertical lines in the Cartesian plane are parallel if and only if they have the same slope.

The phrase "if and only if" in this theorem means two things:
1) If two non-vertical lines in the Cartesian plane are parallel, then they have the same slope.
2) If two non-vertical lines in the Cartesian plane lines have the same slope, then they are parallel.

Objective 2: Understanding the Definition of Perpendicular Lines

If two *distinct* lines are not parallel, then they must intersect at a single point. If the two lines intersect at a right angle (90°), the lines are said to be **perpendicular**.

Theorem: Two non-vertical lines in the Cartesian plane are perpendicular if and only if the product of their slopes is -1.

The theorem above states that if line 1 with slope m_1 is perpendicular to line 2 with slope m_2 then $m_1 m_2 = -1$. Since the product of the slopes of non-vertical perpendicular lines is -1, it follows that $m_1 = -\frac{1}{m_2}$. This means that if $m_1 = \frac{a}{b}$, then $m_2 = -\frac{b}{a}$. In other words, the slopes of non-vertical perpendicular lines are **negative reciprocals** of each other.

SUMMARY OF PARALLEL AND PERPENDICULAR LINES

Given two non-vertical lines l_1 and l_2 such that the slope of line l_1 is $m_1 = \frac{a}{b}$,

1. l_2 is parallel to l_1 if and only if $m_2 = \frac{a}{b}$.
2. l_2 is perpendicular to l_1 if and only if $m_2 = -\frac{b}{a}$. (If l_1 is a horizontal line with slope 0, then l_2 is a vertical line with undefined slope.)

Note that any two distinct vertical lines are parallel to each other while any horizontal line is perpendicular to any vertical line.

Objective 3: Determine if Two Lines are Parallel, Perpendicular, or Neither

Given any two distinct lines, we are now able to quickly determine whether the lines are parallel to each other, perpendicular to each other or neither by simply evaluating their slopes.

Objective 4: Finding the Equations of Parallel and Perpendicular Lines

Follow the procedures from Section 2.3 using the relationships between the slopes of parallel and perpendicular lines.