Sunrise on Mercury

Christina Crow, Emily Tarvin & Kevin Bowman

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- This great speed causes Mercury to have a very short year; only 87.9 Earth days.
- A solar day on Mercury lasts 2 Mercurian years, or 176 Earth days, and a sidereal day lasts 58.6 Earth days, or $\frac{2}{3}$ of a Mercurian year.

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- The purpose of this project is to explore, explain, and illustrate this unique phenomenon.

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- All three of these laws were considered throughout this project.

Useful Formulas For Ellipses

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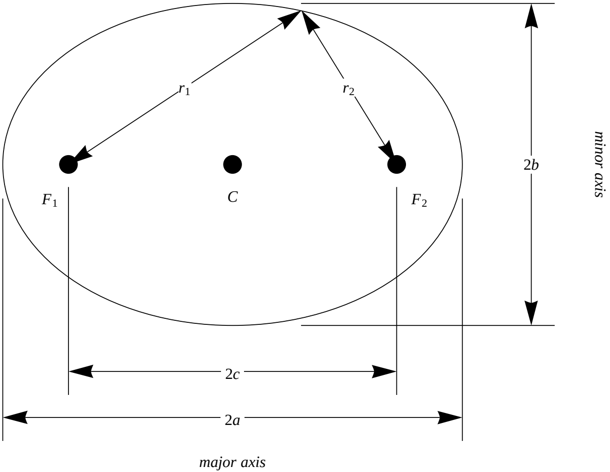
- An ellipse can be described by the equation

$$r_1 + r_2 = 2a$$

where r_1 and r_2 are the distances from both foci to a corresponding point on the curve.

Useful Formulas For Ellipses



This graphic was obtained from
http://mathworld.wolfram.com/Ellipse.html

Useful Formulas For Ellipses

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- The ellipse can be expressed in polar coordinates $x = a\cos(\phi)$ and $y = b\sin(\phi)$ for some parameter, ϕ.

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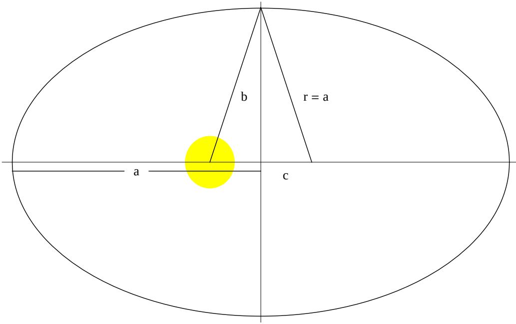
- The area of an ellipse can be expressed as $A = \pi ab$.

Derivation of Semi-Major Axis, Semi-Minor Axis, and Distance from Center to Focus

- In order to find the values of a, b, and c unique to Mercury's elliptical orbit, we derived the equations based on aphelion and perihelion.
- We observe that the semi-major axis is described by

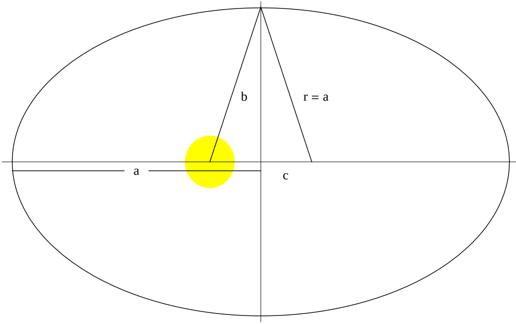
$$a = \frac{aphelion + perihelion}{2}$$



Derivation of Semi-Major Axis, Semi-Minor Axis, and Distance from Center to Focus

It is also apparent that the distance from the center to a focus is described by

$$c = a - perihelion$$



Derivation of Semi-Major Axis, Semi-Minor Axis, and Distance from Center to Focus

- To find the length of the semi-minor axis, we must use the formula

$$r_1 + r_2 = 2a$$

and set $r_1 = r_2$.

- Hence, $r = a$. Using the Pythagorean Theorem, we find that

$$b = \sqrt{a^2 - c^2}$$

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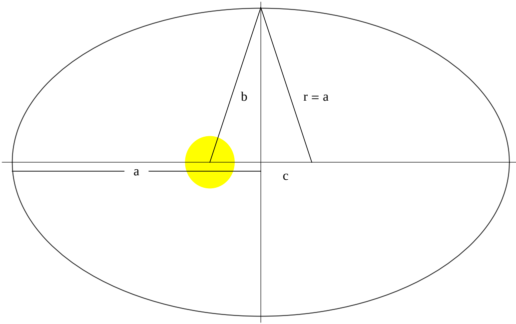

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- We are also able derive a, b, and c in terms of eccentricity.
- Since eccentricity is defined as $\frac{c}{a}$ and $c = \sqrt{a^2 - b^2}$, we rewrite eccentricity as $\frac{\sqrt{a^2-b^2}}{a}$ and solve for b to obtain

$$b = a\sqrt{1 - eccentricity^2}$$

Using Area to Obtain a Function of Time

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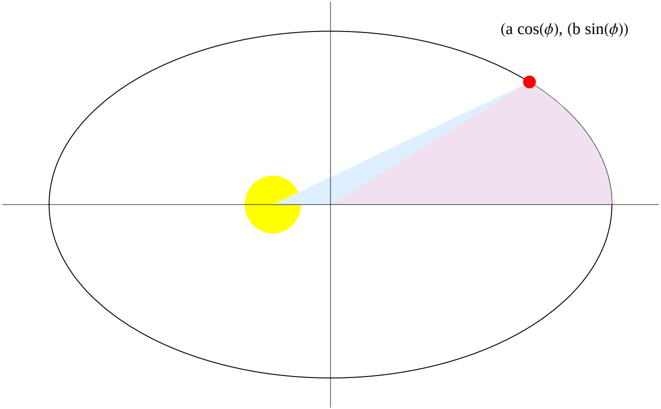
- We can use area as a measure of time since they are directly proportional.

Using Area to Obtain a Function of Time

The function of position that gives time is demonstrated by the equation

$$A(\phi) = \frac{1}{2}(cb\sin(\phi) + ab\phi)$$



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- The new equation is

$$A(\phi) = \frac{1}{2ab\pi}(cb\sin(\phi) + ab\phi)$$

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- Now that we have a function of the area that gives time, we needed to invert this equation to obtain a function of time that gives position.
- We let *Mathematica* compute this for us. We created an animation that uses the inverted function to show Mercury orbiting the Sun and demonstrates the increased speed of Mercury at perihelion as well as the decreased speed at aphelion.

Using Vectors to Track the Sun's Position

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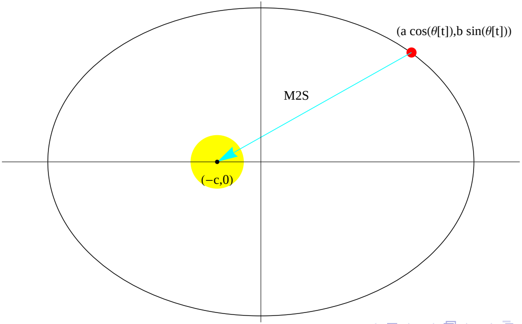


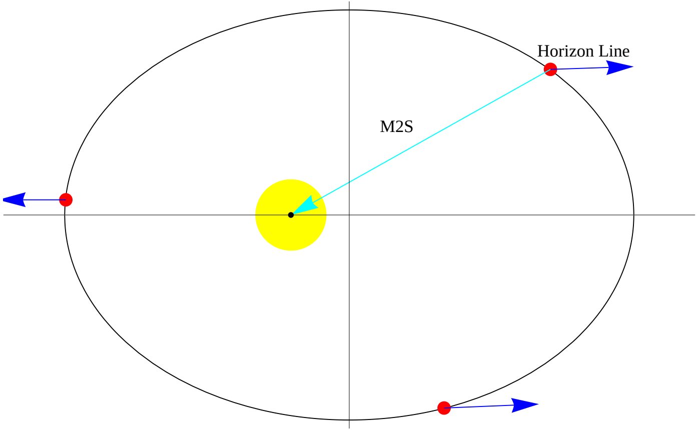
Using Vectors to Track the Sun's Position

- Suppose γ is the angle of Mercury's rotation.
- Since Mercury orbits twice for every three rotations, the angle-to-area ratio is $\frac{2\pi}{\frac{2}{3}} = 3\pi$.
- This implies that in time t, Mercury will have rotated $\gamma = 3\pi t + \gamma_0$ where γ_0 is the initial angle.
- Hence, the vector for Mercury's horizon line is

$$(\cos(3\pi t + \gamma_0), \sin(3\pi t + \gamma_0))$$

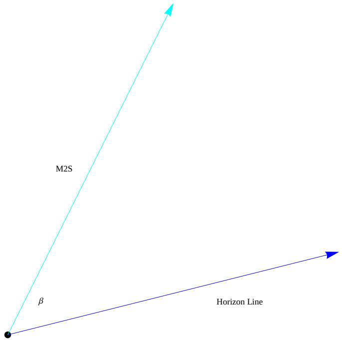
.

Using Vectors to Track the Sun's Position



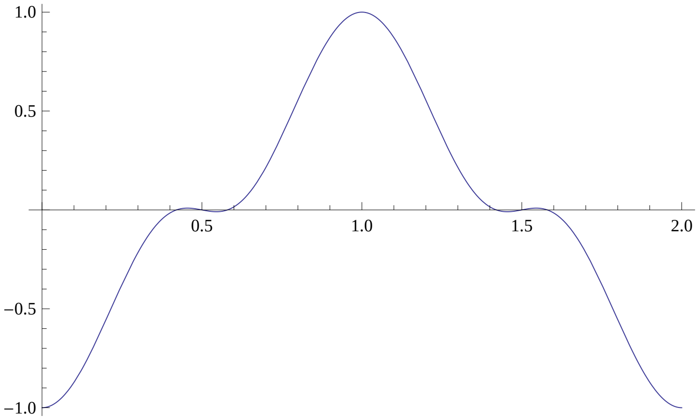
Using Vectors to Track the Sun's Position

- In order to find the angle between the vector representing the horizon line and the vector from Mercury to the Sun, we used the definition of the dot product and cross product.
- We normalized both of these vectors so that their magnitude is 1.
- $\cos(\beta) = \frac{MercurytoSun \cdot HorizonLine}{||MercurytoSun||||HorizonLine||}$
- $\sin(\beta) = \frac{MercurytoSun \times HorizonLine}{||MercurytoSun||||HorizonLine||}$



The Unique Phenomenon

The graph represented by the sine function is



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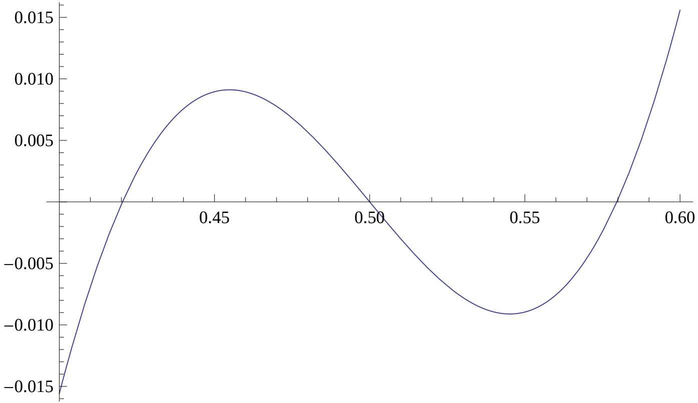
Zoomed in from 0.4 to 0.6:



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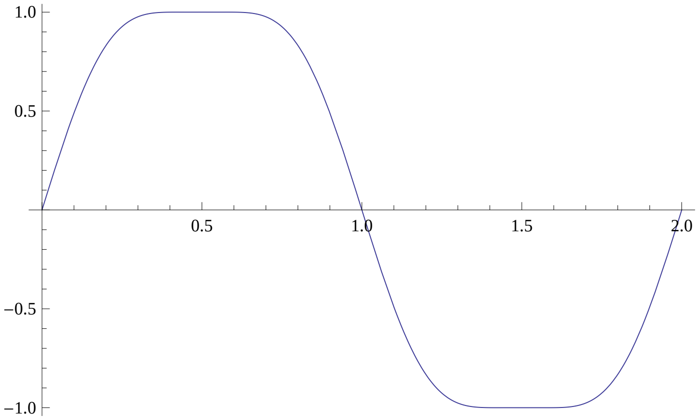
The graph represented by the cosine function is



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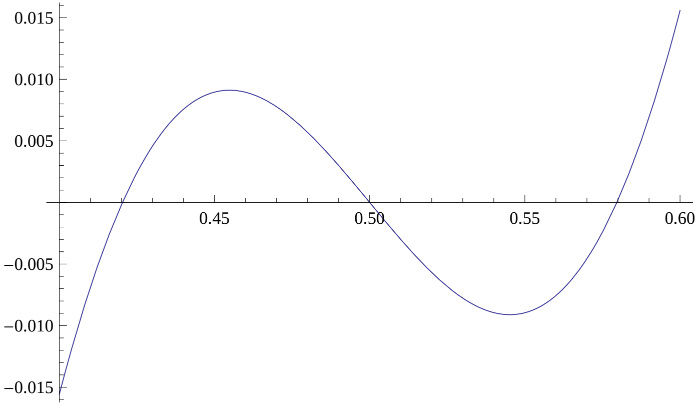
- Both of the functions "dip" at these values for t.
- How does all of this apply to Mercury's sunrise and sunset?

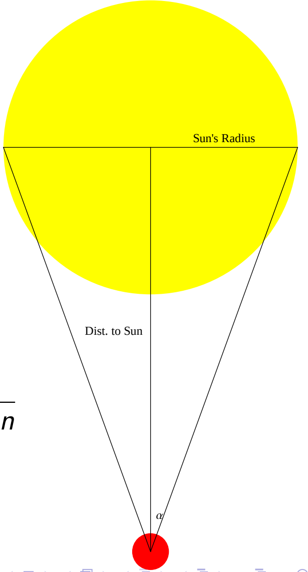
The Unique Phenomenon



Miscellaneous

- A change in the Sun's size can be easily detected on Mercury.
- To understand the perspective of the Sun from Mercury, we can find the angle, α:

$$\alpha = \tan^{-1} \frac{RadiusoftheSun}{DistancefromMercurytotheSun}$$



Conclusion

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- As a result, an observer on Mercury could witness a double sunrise during a single perihelion passage, and a double sunset during the next perihelion passage which would occur during the same solar day.

Acknowledgements

- Richard
- Dr. Smolinsky
- SMILE coordinators

References

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