

## Introduction

Softball is a sport where the defending team pitches balls to the offensive team who tries to hit the ball with a bat. When the offensive team's *batter* hits the ball, they run to the first of three bases. When a runner has run to all bases and back to where they started, they have scored a point for their team (called a *run*). The defending team's goal is to catch 3 of the batted balls in the air, get the batted ball to a base that a batter is running to, or to strike out 3 of the batters when they are batting.

The main focus of this project is deciding the optimal placement of the defending team's fielders; this is decided by finding the position that players are most likely to catch a ball in the air, either resulting in an out or getting to the ball quickest and throwing it to a base to get a batter out. This idea is called *shifting*. Deciding how to shift the outfielders to get the most outs and the least runs is a huge motivation in both baseball and softball.

## Objectives

The big question for our project is: If I know how a batter hits the ball, where is the best place to place outfielders to increase the likelihood of getting the batter out?

From this main question we get several smaller questions that need to be addressed.

1. How will a given batter hit the ball?
2. What factors affect where the ball will go?
3. Does the handedness of the batter, play a significant role in the direction of the ball?
4. Would a standard positioning ever work better than a shifted positioning?

All of these questions are crucial to addressing a given shifting model.

## Standard Positioning

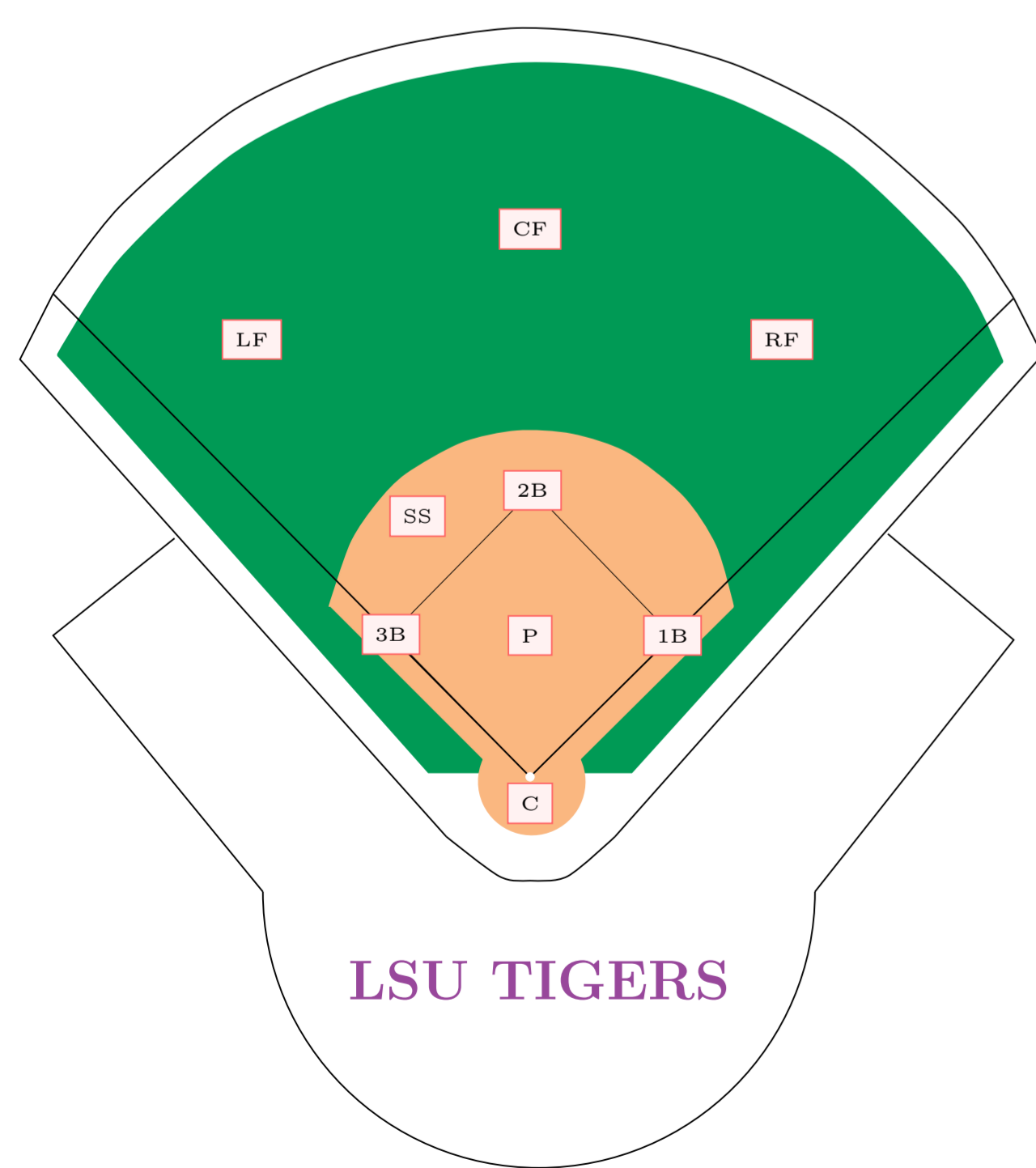


Figure 1. Standard Position of Fielders

In softball, there is an understood standard position for the outfielders. This standard position is optimal, in that every fielder has a region of the field that is 'theirs'. They are responsible for the balls that land in that region. As seen in figure 1, the outfielders are spread far away from each other, and certain plays will require more effort from some players than others. By shifting, the responsibility is less concentrated on a single outfielder.

In the standard position, shown in figure 1, the left, center, and right fielders are represented by LF, CF, and RF. The shortstop is represented by SS, the basemen are represented by their base and the letter B. The catcher and pitcher are represented by C and P.

This standard positioning would not be optimal if, for example, the fielders knew that a given batter tends to hit towards left field. In this case, the right fielder could move closer to center field and this shifting would improve the probability of catching a fly ball.

## Addressing Big Questions

When trying to find the most optimal placement of defenders on the field, one should consider a few different options. Runners could be on any combination of first, second, and third bases or no bases at all. These different combinations of placement of our runners will also affect the placement of our fielders.

In addition to the placement of the runners, one should consider where balls are likely to be hit. If we know that a batter is left handed, then we would expect them to hit the ball more towards right field. With this in mind, it was determined that the best course of action would be to apply a zero-sum game model along with a k-means clustering algorithm to find the most optimal placement of our defenders.

## Zero-Sum Game

The *Zero-Sum Game* is a type of game from game theory. In this game, there are two players and the win of one player is the loss of the other, or vice versa. There is no notion of a tie or draw. We have a Nash Equilibrium (also called a Min-max equilibrium) where we can minimize the maximum winnings of one player and maximize the minimum winnings of the other. In softball, this looks like the batter (*b*) trying to maximize their minimum run production ( $E_R$ ), while the defending team (*d*) tries to minimize the maximum number of runs produced. The equation for the Min-max equation is as follows.

$$\max_b \min_d E_R(b, d) = \min_d \max_b E_R(b, d)$$

This game theoretic method when paired with the k-means clustering algorithm (for finding the center of a given cluster), provides us a way to finding the optimal placement of fielders for our team.

## Results

In the results of the zero-sum game and min-max optimization, there is a clear pattern for the placement of the defending fielders. When the batter is left-handed, it is expected that they are more likely to hit the ball to the right field and vice versa. This can be seen in the spray charts below that outline where to place fielders. When the batter is left (resp. right) handed, the spray charts tell us to place the outfielders in the right (left) field, compared to the standard position outlined in figure 1. Each of the following charts is based on whether the batter has made it to a given base.

Below are the positioning charts for the defense team when the left-handed and right-handed batters have reached a given base.

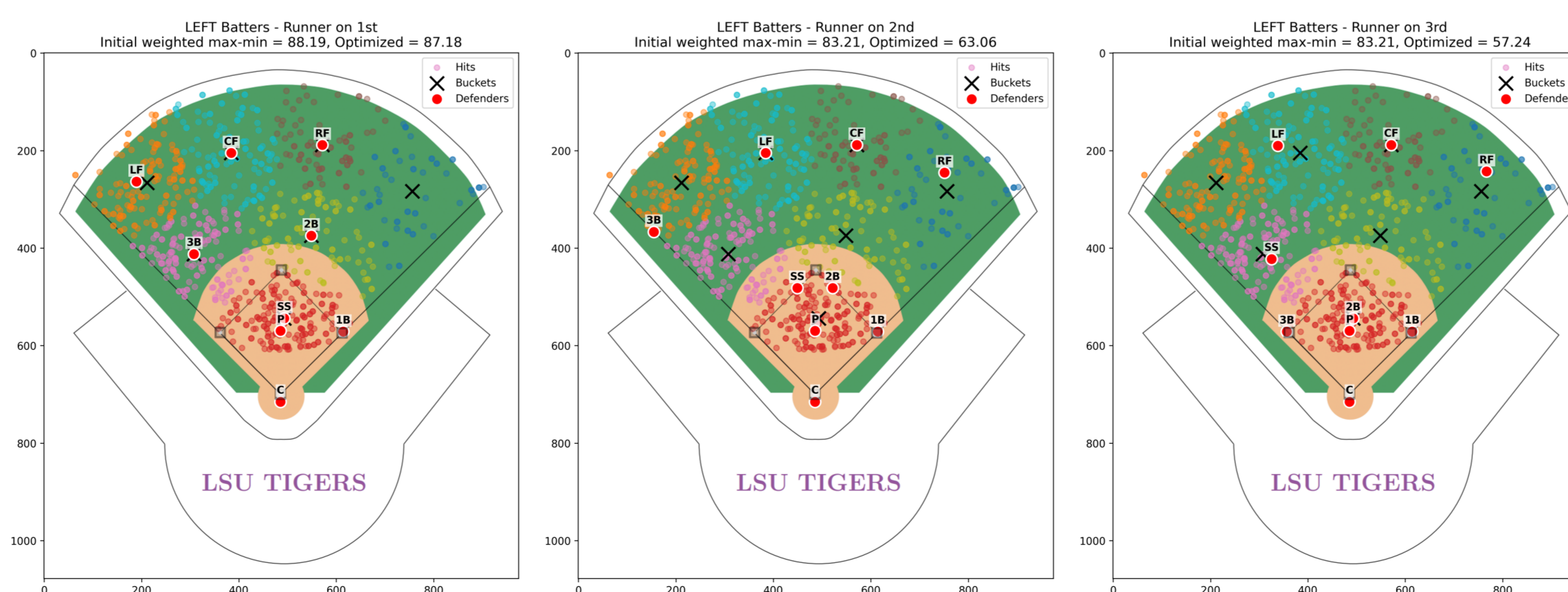


Figure 2. Defense Strategies for Left-hand Runners shown when they are on different bases

Each panel displays spray-chart data (colored points), k-means cluster centroids (black x), and optimized defender positions (red dots) obtained from the weighted min-max formulation.

## Results

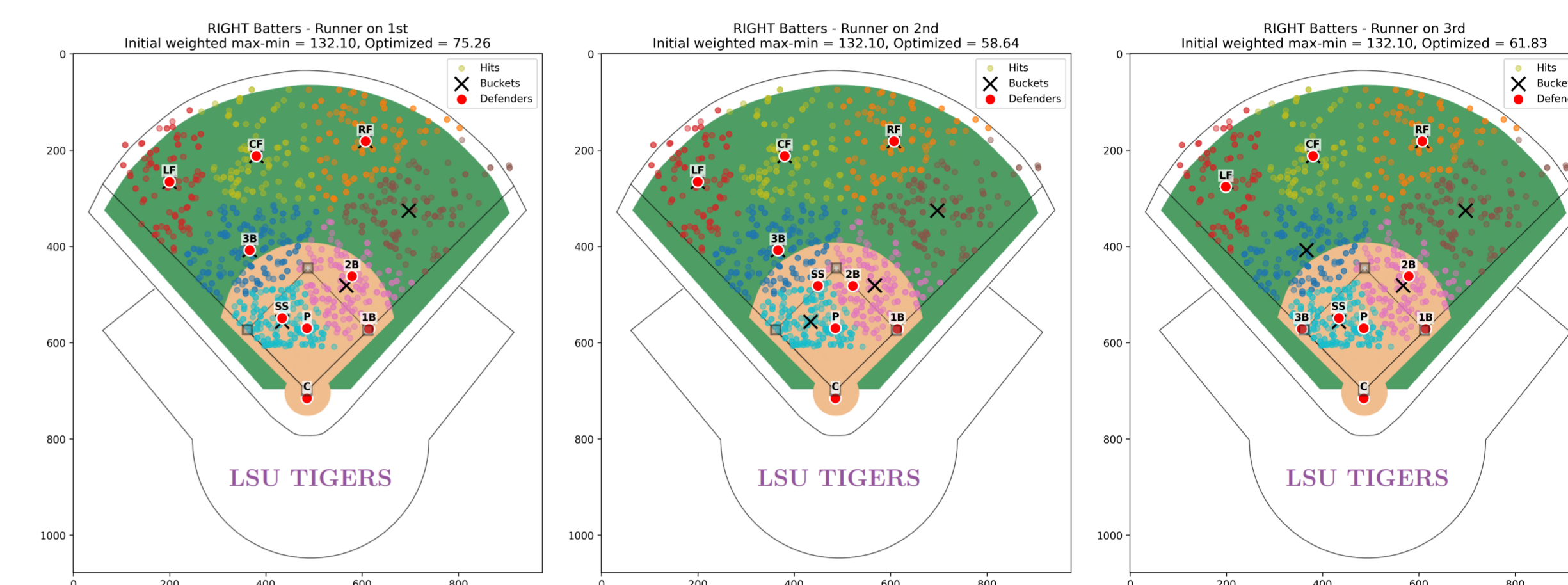


Figure 3. Defense Strategies for Right-hand Runners shown when they are on different bases

The title of each subplot reports the initial and optimized values, where lower values indicate improved defensive coverage and reduced expected run impact. For left-handed batters (top row), defenders shift toward right field, while for right-handed batters (bottom row), the shift moves toward left field, reflecting opposite pull tendencies. As the runner advances from first to third, the infield compresses toward the bases and home plate to prevent run-scoring plays, while the outfield adjusts to maintain coverage of high-density hit regions.

## Moving Forward

The techniques used here can be applied to help with improving the softball team's defensive capabilities. Shifting based on the handedness of the batters can result in more outs for the defense and less runs for the offense. In future work, there are several directions that this project can take: (1) More emphasis on which teams are batting. This could provide tailored plays that would be helpful against specific teams. (2) Greater focus on how different shifting models affect the game (from a game theoretic stand-point). With this direction, a question to ask is, "Does shifting give more vulnerability in other areas of the field that an offensive player could take advantage of?" In this project, an assumption was that if a batter is left/right handed, then they will hit the ball more to the right/left side of the field. (3) Categorizing non-LSU batters into distinct hitter profiles based on their batted ball tendencies and apply the same min-max optimization framework to each profile. This would develop more precise defensive strategies that reduce expected runs. (4) Exploring HDBSCAN and Gaussian Mixture Models as alternatives to k-means to better capture nonlinear patterns in sports analytics.

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## References

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