# On the Prime Number Subset of the Fibonacci Numbers

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# What is a sieve?

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A sieve is a method to count or estimate the size of “sifted sets” of integers. Well, what is a sifted set? A sifted set is made of the remaining numbers after filtering.

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# History

## Two Famous and Useful Sieves

- Sieve of Eratosthenes
- Brun’s Sieve

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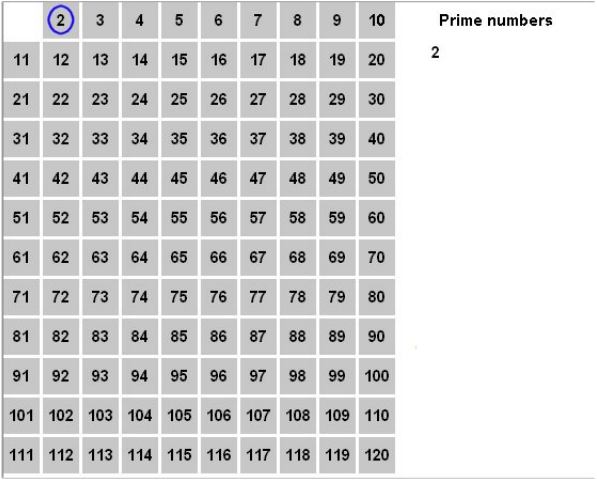
The Sieve of Eratosthenes

## Basic Definitions



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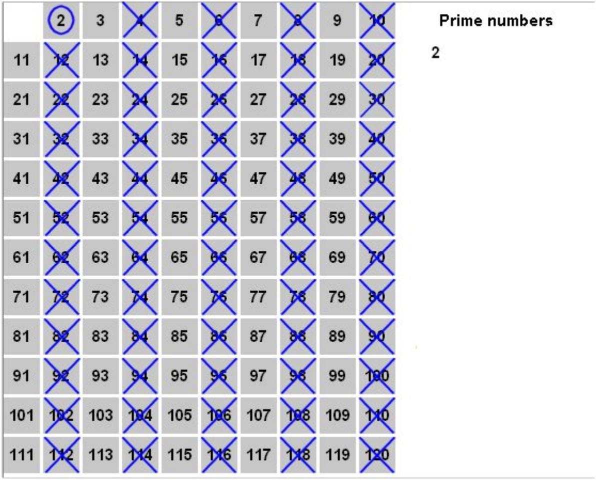
**Prime numbers**

2



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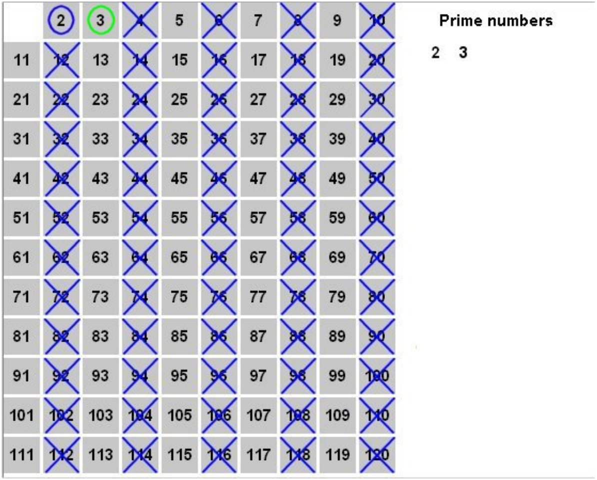
**Prime numbers**

2

## Basic Definitions



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**Prime numbers**

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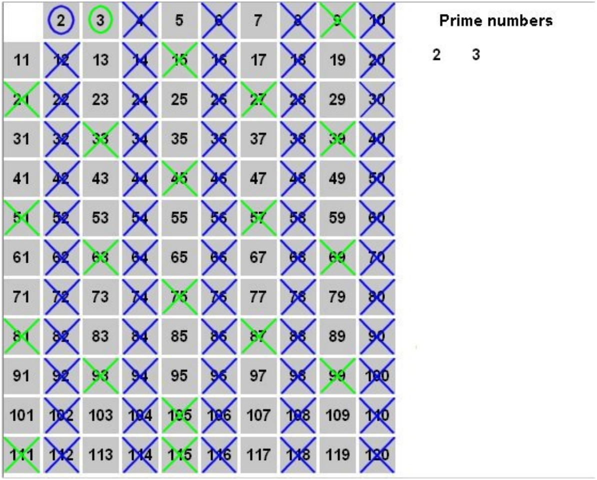
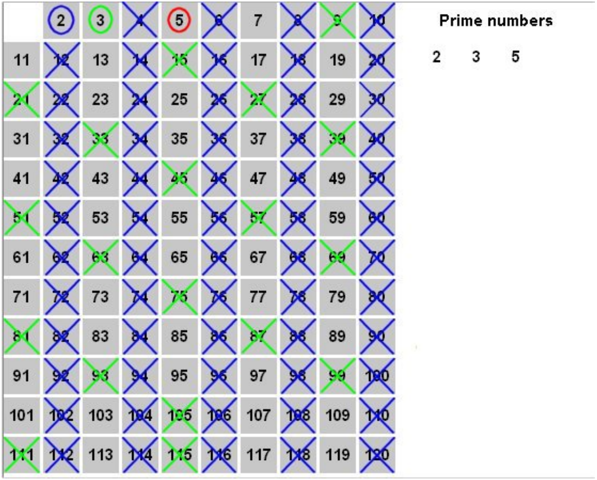

Prime numbers
2
3



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**Prime numbers**

2 3 5

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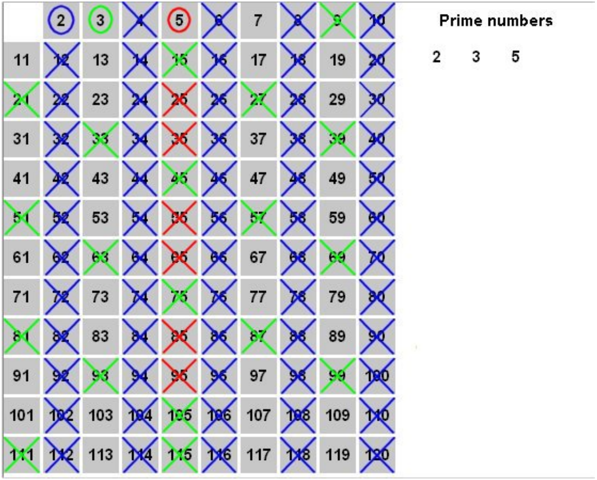

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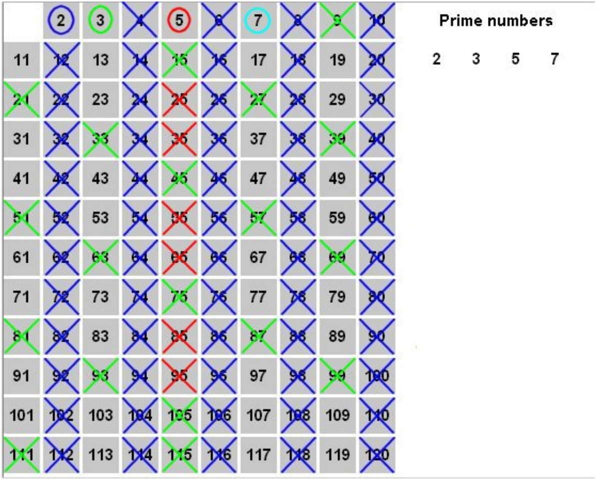

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**Prime numbers**

2 3 5 7

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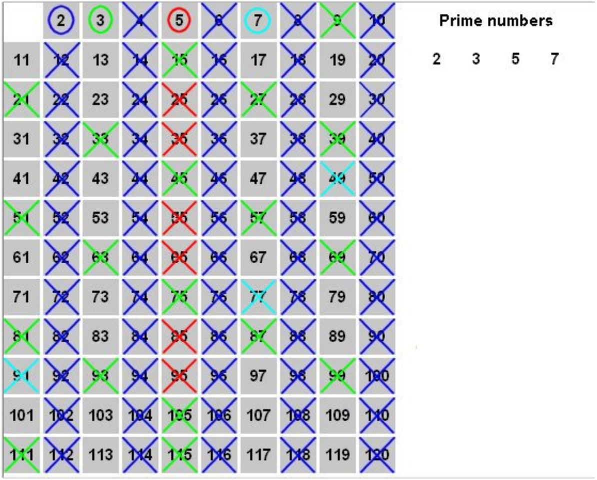

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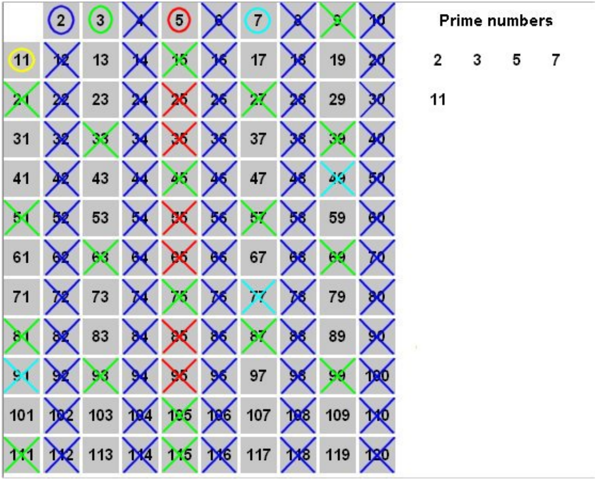

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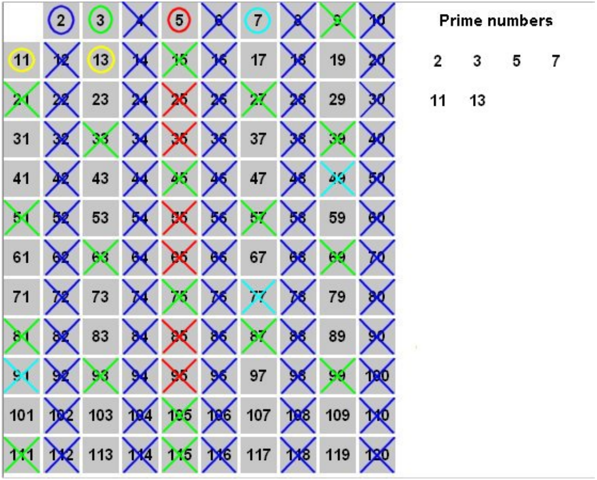
11



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**Prime numbers**

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11 13

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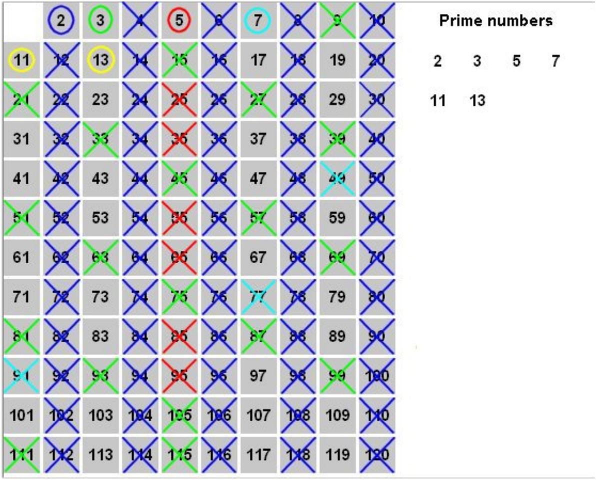

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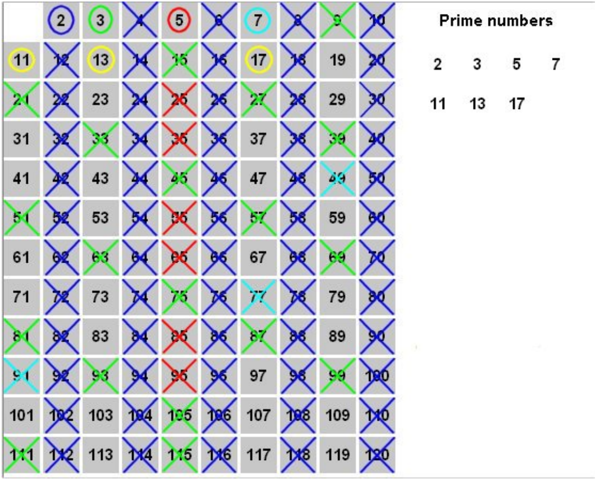

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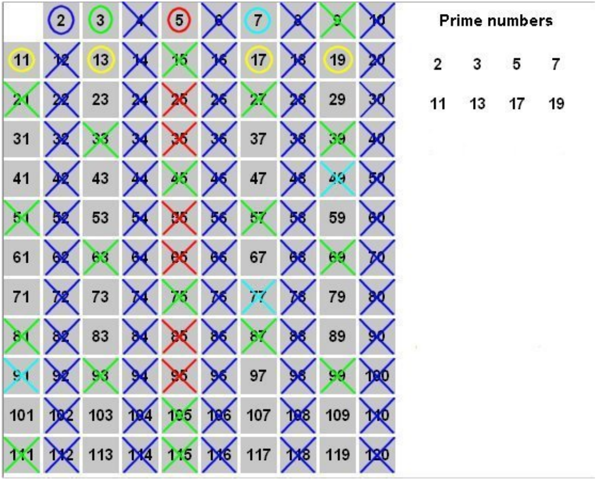

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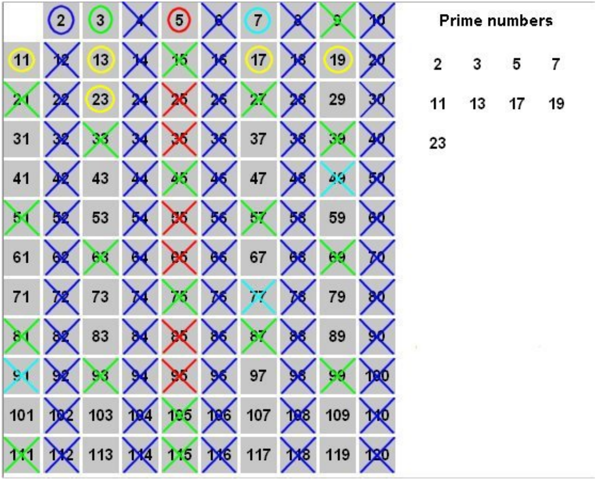

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23

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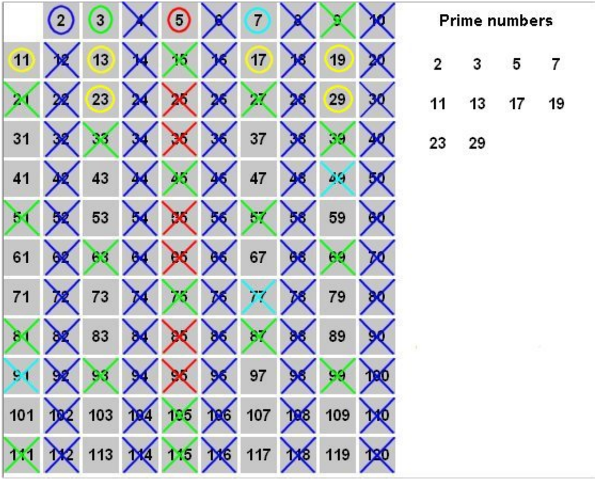

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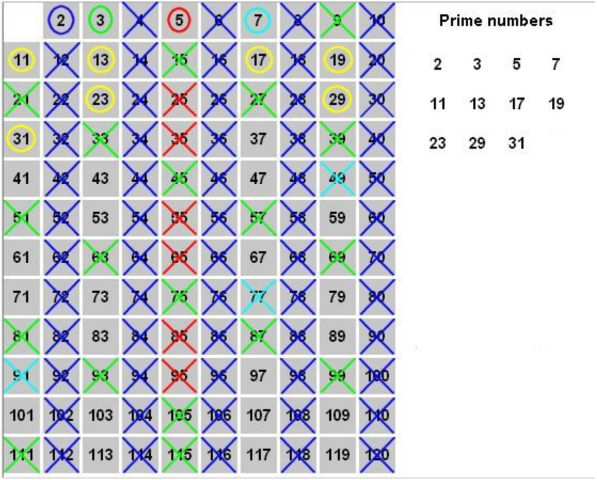
| 101 | ~~102~~ | 103 | ~~104~~ | ~~105~~ | ~~106~~ | 107 | ~~108~~ | 109 | ~~110~~ |
| ~~111~~ | ~~112~~ | 113 | ~~114~~ | ~~115~~ | ~~116~~ | 117 | ~~118~~ | 119 | ~~120~~ |

**Prime numbers**

2 3 5 7
11 13 17 19
23 29

## Basic Definitions



| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | ~~4~~ | 5 | ~~6~~ | 7 | ~~8~~ | ~~9~~ | ~~10~~ |
| 11 | ~~12~~ | 13 | ~~14~~ | ~~15~~ | ~~16~~ | 17 | ~~18~~ | 19 | ~~20~~ |
| ~~21~~ | ~~22~~ | 23 | ~~24~~ | ~~25~~ | ~~26~~ | ~~27~~ | ~~28~~ | 29 | ~~30~~ |
| 31 | ~~32~~ | ~~33~~ | ~~34~~ | ~~35~~ | ~~36~~ | 37 | ~~38~~ | ~~39~~ | ~~40~~ |
| 41 | ~~42~~ | 43 | ~~44~~ | ~~45~~ | ~~46~~ | 47 | ~~48~~ | ~~49~~ | ~~50~~ |
| ~~51~~ | ~~52~~ | 53 | ~~54~~ | ~~55~~ | ~~56~~ | ~~57~~ | ~~58~~ | 59 | ~~60~~ |
| 61 | ~~62~~ | ~~63~~ | ~~64~~ | ~~65~~ | ~~66~~ | 67 | ~~68~~ | ~~69~~ | ~~70~~ |
| 71 | ~~72~~ | 73 | ~~74~~ | ~~75~~ | ~~76~~ | ~~77~~ | ~~78~~ | 79 | ~~80~~ |
| ~~81~~ | ~~82~~ | 83 | ~~84~~ | ~~85~~ | ~~86~~ | ~~87~~ | ~~88~~ | 89 | ~~90~~ |
| ~~91~~ | ~~92~~ | ~~93~~ | ~~94~~ | ~~95~~ | ~~96~~ | 97 | ~~98~~ | ~~99~~ | ~~100~~ |
| 101 | ~~102~~ | 103 | ~~104~~ | ~~105~~ | ~~106~~ | 107 | ~~108~~ | 109 | ~~110~~ |
| ~~111~~ | ~~112~~ | 113 | ~~114~~ | ~~115~~ | ~~116~~ | 117 | ~~118~~ | 119 | ~~120~~ |

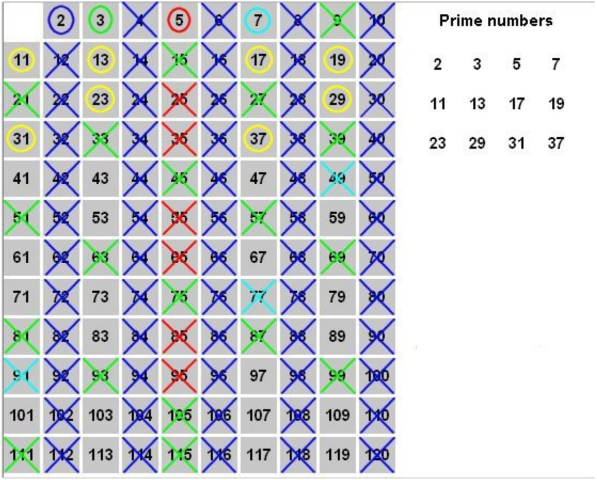
**Prime numbers**

2 3 5 7

11 13 17 19

23 29 31

## Basic Definitions



| | | | | | | | | | |
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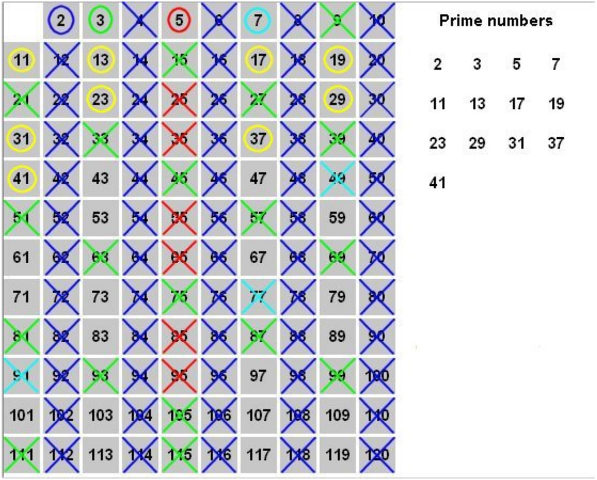
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

**Prime numbers**

| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 11 | 13 | 17 | 19 |
| 23 | 29 | 31 | 37 |



| | | | | | | | | | |
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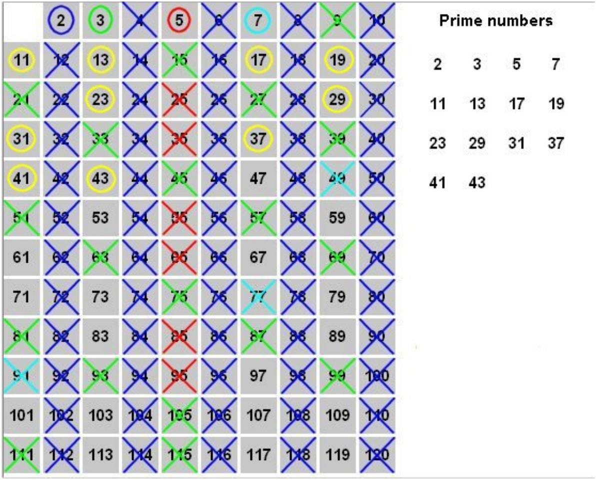
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

**Prime numbers**

| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 11 | 13 | 17 | 19 |
| 23 | 29 | 31 | 37 |
| 41 | | | |



| | | | | | | | | | |
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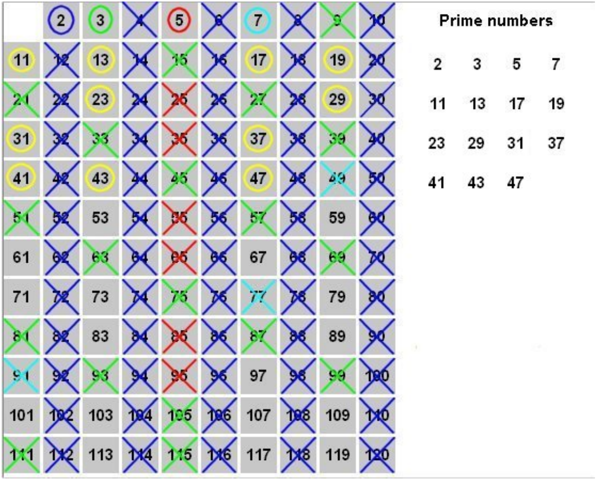
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

**Prime numbers**

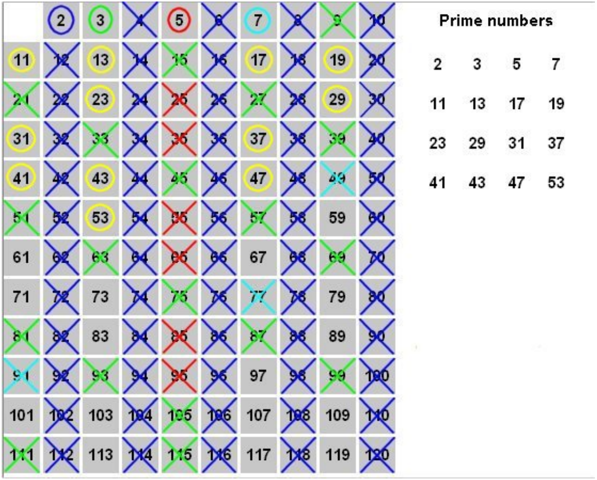
| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 11 | 13 | 17 | 19 |
| 23 | 29 | 31 | 37 |
| 41 | 43 | | |



| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

**Prime numbers**

| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 11 | 13 | 17 | 19 |
| 23 | 29 | 31 | 37 |
| 41 | 43 | 47 | |



| | | | | | | | | | |
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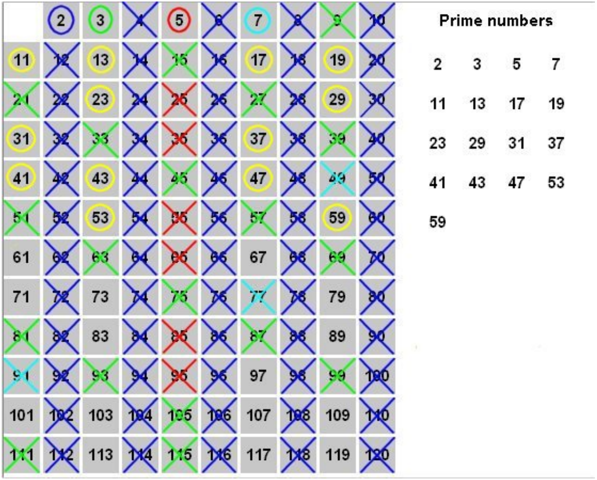
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

**Prime numbers**

| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 11 | 13 | 17 | 19 |
| 23 | 29 | 31 | 37 |
| 41 | 43 | 47 | 53 |

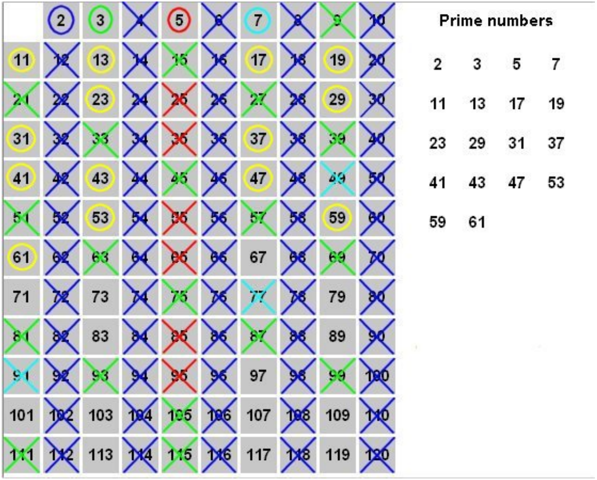


| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | ~~4~~ | 5 | ~~6~~ | 7 | ~~8~~ | ~~9~~ | ~~10~~ |
| 11 | ~~12~~ | 13 | ~~14~~ | ~~15~~ | ~~16~~ | 17 | ~~18~~ | 19 | ~~20~~ |
| ~~21~~ | ~~22~~ | 23 | ~~24~~ | ~~25~~ | ~~26~~ | ~~27~~ | ~~28~~ | 29 | ~~30~~ |
| 31 | ~~32~~ | ~~33~~ | ~~34~~ | ~~35~~ | ~~36~~ | 37 | ~~38~~ | ~~39~~ | ~~40~~ |
| 41 | ~~42~~ | 43 | ~~44~~ | ~~45~~ | ~~46~~ | 47 | ~~48~~ | ~~49~~ | ~~50~~ |
| ~~51~~ | ~~52~~ | 53 | ~~54~~ | ~~55~~ | ~~56~~ | ~~57~~ | ~~58~~ | 59 | ~~60~~ |
| 61 | ~~62~~ | ~~63~~ | ~~64~~ | ~~65~~ | ~~66~~ | 67 | ~~68~~ | ~~69~~ | ~~70~~ |
| 71 | ~~72~~ | 73 | ~~74~~ | ~~75~~ | ~~76~~ | ~~77~~ | ~~78~~ | 79 | ~~80~~ |
| ~~81~~ | ~~82~~ | 83 | ~~84~~ | ~~85~~ | ~~86~~ | ~~87~~ | ~~88~~ | 89 | ~~90~~ |
| ~~91~~ | ~~92~~ | ~~93~~ | ~~94~~ | ~~95~~ | ~~96~~ | 97 | ~~98~~ | ~~99~~ | ~~100~~ |
| 101 | ~~102~~ | 103 | ~~104~~ | ~~105~~ | ~~106~~ | 107 | ~~108~~ | 109 | ~~110~~ |
| ~~111~~ | ~~112~~ | 113 | ~~114~~ | ~~115~~ | ~~116~~ | 117 | ~~118~~ | 119 | ~~120~~ |

**Prime numbers**

| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 11 | 13 | 17 | 19 |
| 23 | 29 | 31 | 37 |
| 41 | 43 | 47 | 53 |
| 59 | | | |

## Basic Definitions



| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

**Prime numbers**

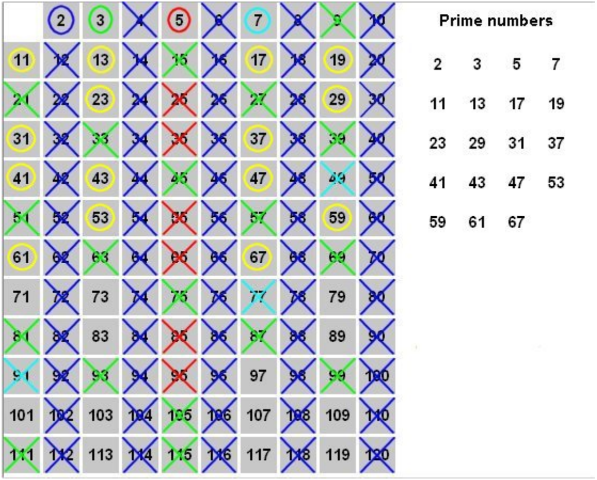
2 3 5 7

11 13 17 19

23 29 31 37

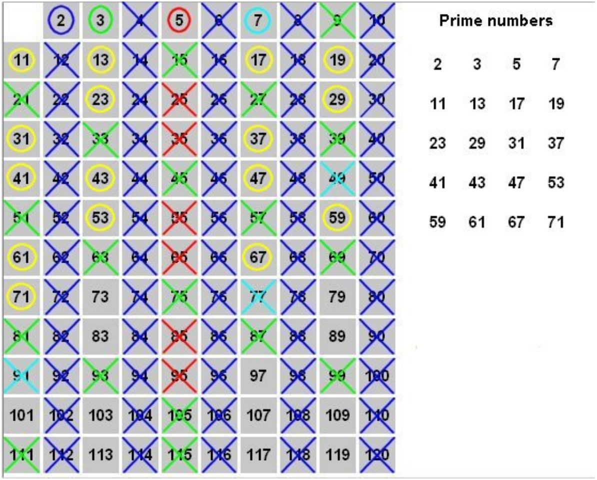
41 43 47 53

59 61



**Prime numbers**

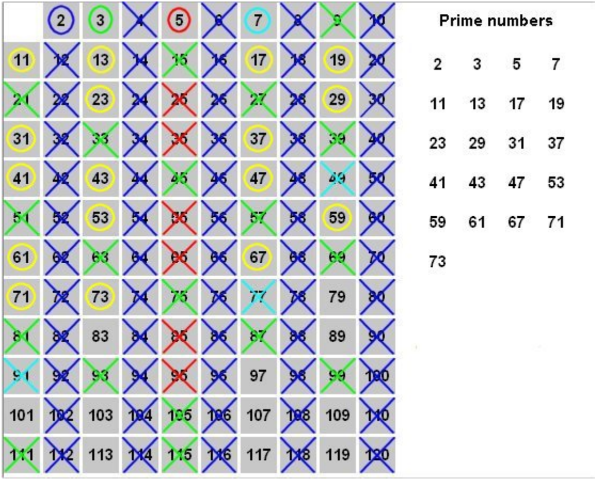
| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 11 | 13 | 17 | 19 |
| 23 | 29 | 31 | 37 |
| 41 | 43 | 47 | 53 |
| 59 | 61 | 67 | |



| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | ~~4~~ | 5 | ~~6~~ | 7 | ~~8~~ | ~~9~~ | ~~10~~ |
| 11 | ~~12~~ | 13 | ~~14~~ | ~~15~~ | ~~16~~ | 17 | ~~18~~ | 19 | ~~20~~ |
| ~~21~~ | ~~22~~ | 23 | ~~24~~ | ~~25~~ | ~~26~~ | ~~27~~ | ~~28~~ | 29 | ~~30~~ |
| 31 | ~~32~~ | ~~33~~ | ~~34~~ | ~~35~~ | ~~36~~ | 37 | ~~38~~ | ~~39~~ | ~~40~~ |
| 41 | ~~42~~ | 43 | ~~44~~ | ~~45~~ | ~~46~~ | 47 | ~~48~~ | ~~49~~ | ~~50~~ |
| ~~51~~ | ~~52~~ | 53 | ~~54~~ | ~~55~~ | ~~56~~ | ~~57~~ | ~~58~~ | 59 | ~~60~~ |
| 61 | ~~62~~ | ~~63~~ | ~~64~~ | ~~65~~ | ~~66~~ | 67 | ~~68~~ | ~~69~~ | ~~70~~ |
| 71 | ~~72~~ | 73 | ~~74~~ | ~~75~~ | ~~76~~ | ~~77~~ | ~~78~~ | 79 | ~~80~~ |
| ~~81~~ | ~~82~~ | 83 | ~~84~~ | ~~85~~ | ~~86~~ | ~~87~~ | ~~88~~ | 89 | ~~90~~ |
| ~~91~~ | ~~92~~ | ~~93~~ | ~~94~~ | ~~95~~ | ~~96~~ | 97 | ~~98~~ | ~~99~~ | ~~100~~ |
| 101 | ~~102~~ | 103 | ~~104~~ | ~~105~~ | ~~106~~ | 107 | ~~108~~ | 109 | ~~110~~ |
| ~~111~~ | ~~112~~ | 113 | ~~114~~ | ~~115~~ | ~~116~~ | 117 | ~~118~~ | 119 | ~~120~~ |

**Prime numbers**

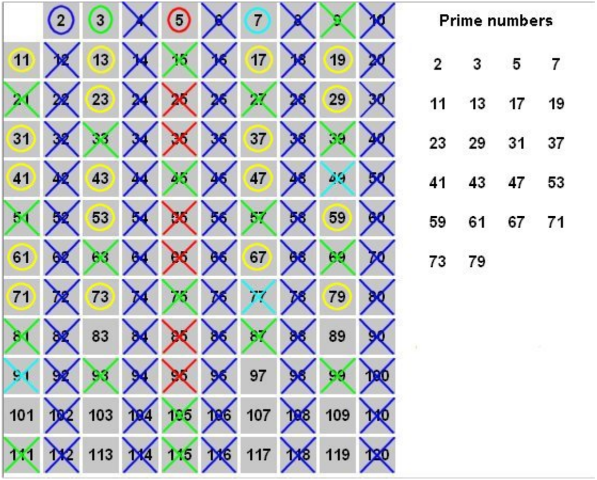
| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 11 | 13 | 17 | 19 |
| 23 | 29 | 31 | 37 |
| 41 | 43 | 47 | 53 |
| 59 | 61 | 67 | 71 |



| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

**Prime numbers**

| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 11 | 13 | 17 | 19 |
| 23 | 29 | 31 | 37 |
| 41 | 43 | 47 | 53 |
| 59 | 61 | 67 | 71 |
| 73 | | | |



| | | | | | | | | | |
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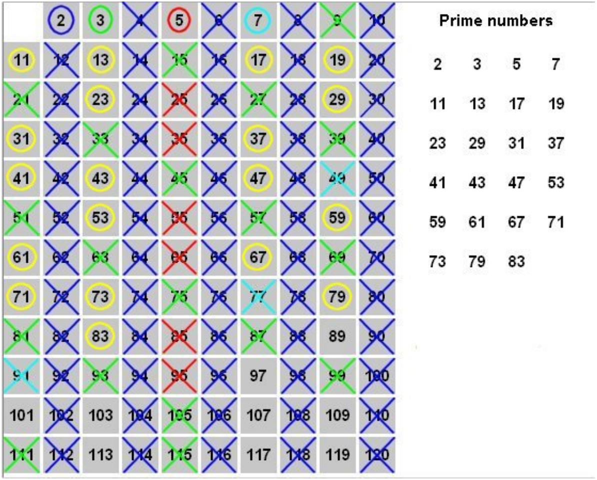
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

**Prime numbers**

2 3 5 7
11 13 17 19
23 29 31 37
41 43 47 53
59 61 67 71
73 79



| | | | | | | | | | |
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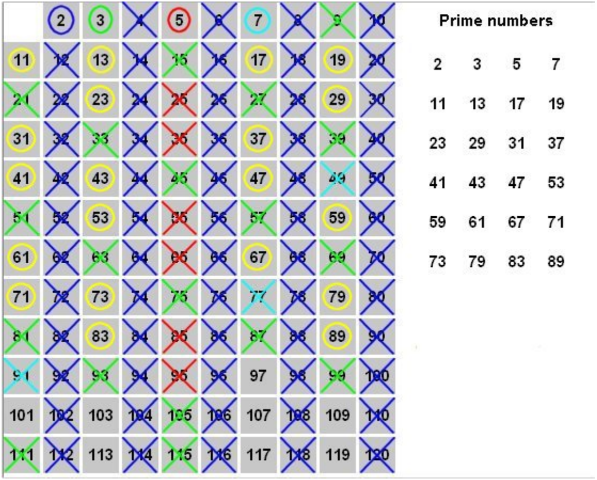
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

**Prime numbers**

| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 11 | 13 | 17 | 19 |
| 23 | 29 | 31 | 37 |
| 41 | 43 | 47 | 53 |
| 59 | 61 | 67 | 71 |
| 73 | 79 | 83 | |



| | | | | | | | | | |
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| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
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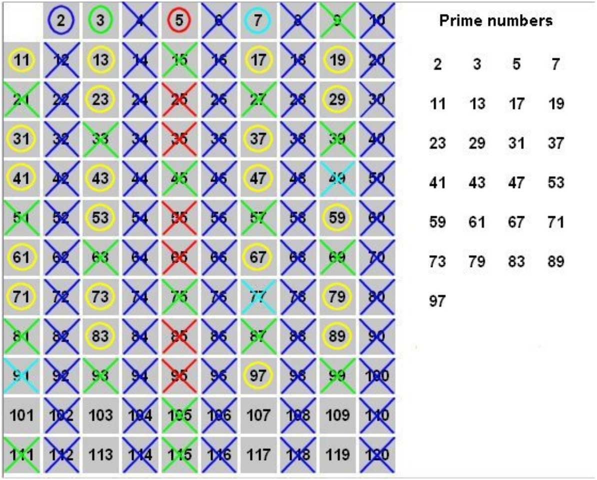
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

**Prime numbers**

| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 11 | 13 | 17 | 19 |
| 23 | 29 | 31 | 37 |
| 41 | 43 | 47 | 53 |
| 59 | 61 | 67 | 71 |
| 73 | 79 | 83 | 89 |

## Basic Definitions



| | | | | | | | | | |
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| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
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| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

**Prime numbers**

| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 11 | 13 | 17 | 19 |
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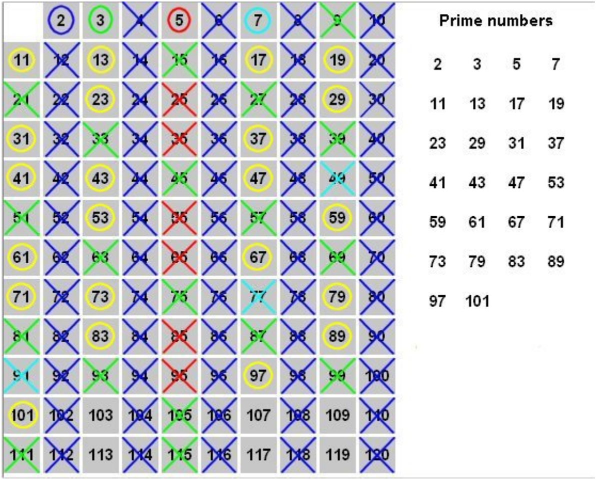
| 41 | 43 | 47 | 53 |
| 59 | 61 | 67 | 71 |
| 73 | 79 | 83 | 89 |
| 97 | | | |



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| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
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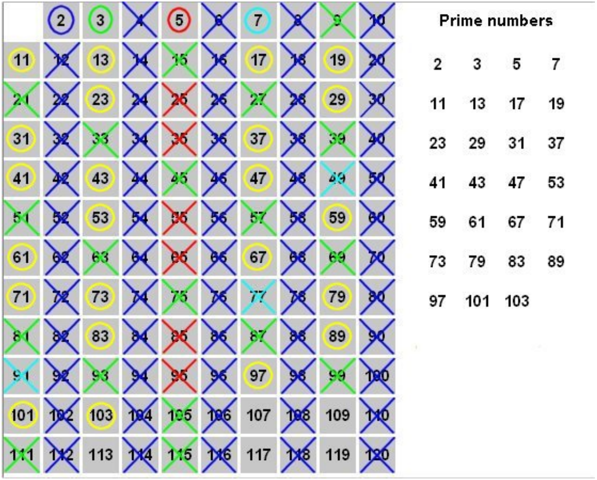
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

**Prime numbers**

| | | | |
|---|---|---|---|
| 2 | 3 | 5 | 7 |
| 11 | 13 | 17 | 19 |
| 23 | 29 | 31 | 37 |
| 41 | 43 | 47 | 53 |
| 59 | 61 | 67 | 71 |
| 73 | 79 | 83 | 89 |
| 97 | 101 | | |



| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |

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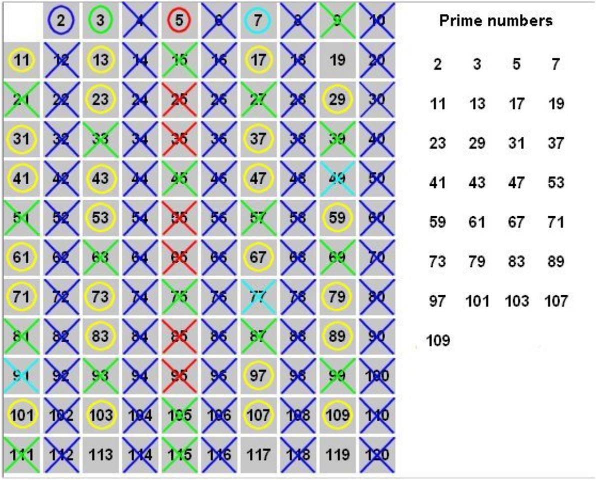

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| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
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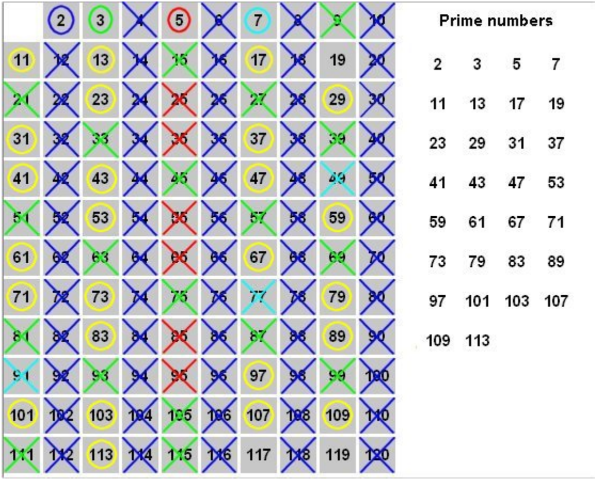

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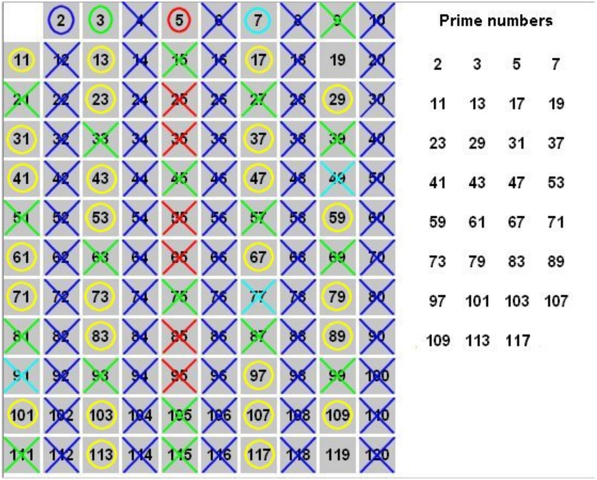

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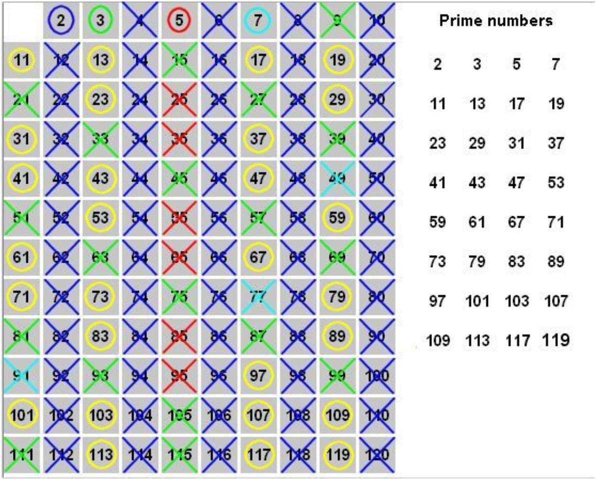

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# Brun Sieve

The Brun sieve is a generalized method compared to the Eratosthenes sieve. It allows us to sieve any set $A$ with a designated set $\mathcal{P}$. It is formally stated as:

$$S(A, \mathcal{P}, z) = |A \setminus \bigcup_{p \in P(z)} A_p|$$

But what does that mean???

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# For clarity, let us look at an example

We may take

$$A = \{5, 6, 10, 11, 12, 13, 18, 20, 22, 24, 28, 35\}$$

and

$$\mathcal{P} = \{2, 7\}.$$

By sifting $A$ with the given $\mathcal{P}$, we see

$$A_2 = \{6, 10, 12, 18, 20, 22, 24, 28\}, \text{ and } A_7 = \{28, 35\}$$

. We are left with $S(A, \mathcal{P}, z) = |\{5, 11, 13\}| = 3.$

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- Goldbach Conjecture

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*The Twin Prime Conjecture states that there are infinitely many primes p such that $p+2$ is also prime. An example is (5,7). This is an unproven conjecture at this point; however, Brun used his sieve to show that the sum of the recipricals converges.*

Brun used his sieve to make progress on the conjecture by showing that there are infinitely many pairs of integers differing by 2, where each of the member of the pair is the product of at most 9 primes.

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# Goldbach Conjecture

This is one of the oldest unsolved problems in mathematics.

**Conjecture**

*Every even integer greater than 2 is a Goldbach number, which is a number that can be expressed as two primes.*

For example:

$$2 + 2 = 4$$
$$3 + 3 = 6$$
$$3 + 5 = 8.$$

Brun used his sieve to make progress on this conjecture as well. He showed that very even number is the sum of two numbers each of which is the product of at most 9 primes.

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# The Famous Fibonacci Sequence

The Fibonacci Sequence is: $F_n$, defined by the recurrence relation:

$$F_n = F_{n-1} + F_{n-2}.$$

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# Brun Sieve and the Fibonacci Sequence

Let us take a finite amount of the Fibonacci sequence.

$$A = F_n = \{2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610\}$$

and let $\mathcal{P} = \{2, 3, 5, 7, 11, ...\}$. After filtering using the set $\mathcal{P}$, the primes, we are left with

$$S(A, \mathcal{P}, z) = |\{2, 3, 5, 13, 89, 233\}| = 6$$

. These are the prime Fibonacci numbers within this given $F_n$.

# Fibonacci Primes

A Fibonacci number that is prime. Their finiteness is unknown. It has been calculated that the largest known Fibonacci prime is $F_{81839}$, which has 17103 digits. It was proven to be such by Broadhurst and de Water in 2001.

# Carmichael’s Theorem

**Theorem**

*Every Fibonacci number (aside from 1, 8, and 144) has at least one unique prime factor that has not been a factor of the preceding Fibonacci numbers.*

0 1 1 2 3 5 8 13 21 34

# Fibonacci sequence

$$\begin{array}{cccccccccc} 0 & 1 & 1 & 2 & 3 & 5 & 8 & 13 & 21 & 34 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0 & 1 & 1 & 2 & 3 & 5 & 1 & 6 & 0 & 6 \end{array}$$

# Fibonacci sequence mod 7

$$0, 1, 1, 2, 3, 5, 1, 6, 0, 6, 6, 5, 4, 2, 6, 1, \underline{0, 1, 1}\ldots$$

**Theorem**

*Let $P$ be an arbitrary finite collection of primes. Then there exists a Fibonacci number that has no factors in $P$.*

# Finding Relative Primes

Modulo 2: zeros every 3rd term

Modulo 3: zeros every 4th term

Modulo 7: zeros every 8th term

24th term: 46368

$= 2 \text{ x } 23184 = 3 \text{ x } 15456 = 7 \text{ x } 6624$

25th term: 75025

$\equiv 1 \ mod\ 2$

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# Distribution of Primes

$$\pi(x) \sim \frac{x}{\log x}$$

$$P_{prime} \approx \frac{\frac{x}{\log x}}{x} = \frac{1}{\log x}$$

# Distribution of Fibonacci Numbers

$$\lim_{x\to\infty} \frac{F_{n+1}}{F_n} = \phi$$

$$F_n \approx c\phi^n$$

$$n \approx \frac{\log x}{\log const.}$$

$$P_{Fibonacci} \approx \frac{\frac{\log x}{const.}}{x}$$

# Probability of Both Prime and Fibonacci

$$P_{prime} \cdot P_{Fibonacci} = \frac{1}{\log x} \cdot \frac{\log x}{x} \cdot const. = \frac{1}{x} \cdot const.$$

# Sum over all natural numbers

$$\sum_x P_{(P \cap F)} \approx \sum_x \frac{1}{x} \to \infty$$

# Sum over all primes

$$\sum_p P_{(P \cap F)} \approx \sum_p \frac{1}{p} \to \infty$$

In conclusion, the Fibonacci primes appear to form an infinite set, but the argument is not valid since the sets are not independent.

The idea of using a matrix was to allow us to easily see the prime Fibonacci numbers. We started off by taking an array with the Fibonacci numbers on top and the primes on the side. Then filled the array with the Fibonacci numbers modulo the primes.

# More Properties

$$\begin{array}{cccccccccccc}
 & 2 & 3 & 5 & 8 & 13 & 21 & 34 & 55 & 89 & \cdots \\
2 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & \\
3 & 2 & 0 & 2 & 2 & 1 & 0 & 1 & 1 & 2 & \\
5 & 2 & 3 & 0 & 3 & 3 & 1 & 4 & 0 & 4 & \\
7 & 2 & 3 & 5 & 1 & 6 & 0 & 6 & 6 & 5 & \\
11 & 2 & 3 & 5 & 8 & 2 & 10 & 1 & 0 & 1 & \\
13 & 2 & 3 & 5 & 8 & 0 & 8 & 8 & 3 & 11 & \\
17 & 2 & 3 & 5 & 8 & 13 & 4 & 0 & 4 & 4 & \\
19 & 2 & 3 & 5 & 8 & 13 & 2 & 15 & 17 & 13 & \\
23 & 2 & 3 & 5 & 8 & 13 & 21 & 11 & 9 & 20 & \\
\vdots & & & & & & & & & & \ddots
\end{array}$$

We then reduced the array so that all the zeros on the diagonal represented the Prime Fibonacci numbers.

$$\begin{array}{c|cccccccccc}
 & 2 & 3 & 5 & 8 & 13 & 21 & 34 & 55 & 89 & \cdots \\
\hline
2 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & \\
3 & 2 & 0 & 2 & 2 & 1 & 0 & 1 & 1 & 2 & \\
5 & 2 & 3 & 0 & 3 & 3 & 1 & 4 & 0 & 4 & \\
11 & 2 & 3 & 5 & 8 & 2 & 10 & 1 & 0 & 1 & \\
13 & 2 & 3 & 5 & 8 & 0 & 8 & 8 & 3 & 11 & \\
29 & 2 & 3 & 5 & 8 & 13 & 21 & 5 & 26 & 2 & \\
37 & 2 & 3 & 5 & 8 & 13 & 21 & 34 & 18 & 15 & \\
59 & 2 & 3 & 5 & 8 & 13 & 21 & 34 & 55 & 30 & \\
89 & 2 & 3 & 5 & 8 & 13 & 21 & 34 & 55 & 0 & \\
\vdots & & & & & & & & & & \ddots
\end{array}$$

Then we created matrices from the array and looked at their properties

$$A3 = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 2 \\ 2 & 3 & 0 \end{pmatrix}$$

$\det(A3) = 10$ $\mathrm{rk}(A3) = 3$

$$A7 = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 2 & 2 & 1 & 0 & 1 \\ 2 & 3 & 0 & 3 & 3 & 1 & 4 \\ 2 & 3 & 0 & 3 & 3 & 1 & 0 \\ 2 & 3 & 5 & 8 & 2 & 10 & 1 \\ 2 & 3 & 5 & 8 & 0 & 8 & 8 \\ 2 & 3 & 5 & 8 & 13 & 21 & 5 \\ 2 & 3 & 5 & 8 & 13 & 21 & 34 \end{pmatrix}$$

$\det(A7) = 0$ $\mathrm{rk}(A7) = 6$

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**Theorem**

*For all* $N \geq 6$, $\det(AN) = 0$.

As it turns out the last row is always a linear combination of the previous rows so,

$RN_N - RN_{N-1} = (0, 0, ...0, a, b)$.

$a \neq 0$ iff $F_N$ or $F_{N-1}$ is a prime.

If the Fibonacci primes are finite
$\exists N$ such that all the Fibonacci primes are $\leq F_N$. Hence
$\forall K > N$,

$$AK = \begin{pmatrix} AN & * \\ * & B \end{pmatrix}$$

Where $TrB = \sum_{i=N+1}^{K} F_i$

### Conjecture

*For all $N \geq 6$, $rk(AN) = N - 1$.*

A partial proof comes from the proof of the theorem. We have that $\text{rk}(AN) \leq N - 1$ for all $N \geq 6$.

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# Thank you