Linear Geometric Constructions

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Introduction

What is a Geometric Construction?

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What is a Geometric Construction?

- Types of Geometric Constructions

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What is a Geometric Construction?

- Types of Geometric Constructions
- Mathematicians

Definitions and Rules for Basic Construcitons

- Tools

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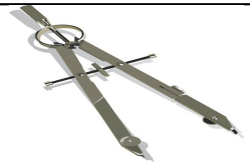
Compass	Straightedge
	

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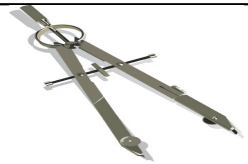	

- Postulates for Basic Constructions

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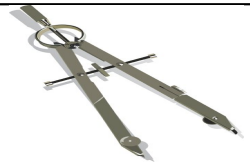	

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  - Assume we can construct two points (the origin and (1,0))

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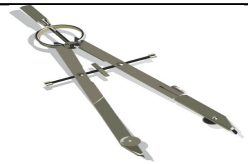	

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  - Constructions of lines and circles

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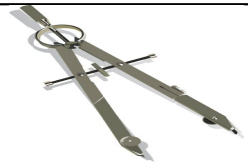	

- Postulates for Basic Constructions
  - Assume we can construct two points (the origin and (1,0))
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  - Use of the intersections of those lines and cirlces to constuct new points

Basic constructions

- Perpendicular Lines

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- Parallel Lines

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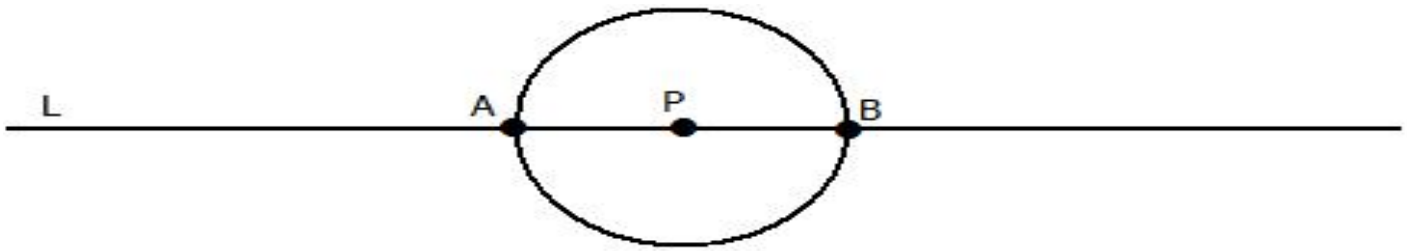
- Parallel Lines
- Squareroots

Basic constructions

- Perpendicular Lines
- Parallel Lines
- Squareroots
- Bisecting an angle

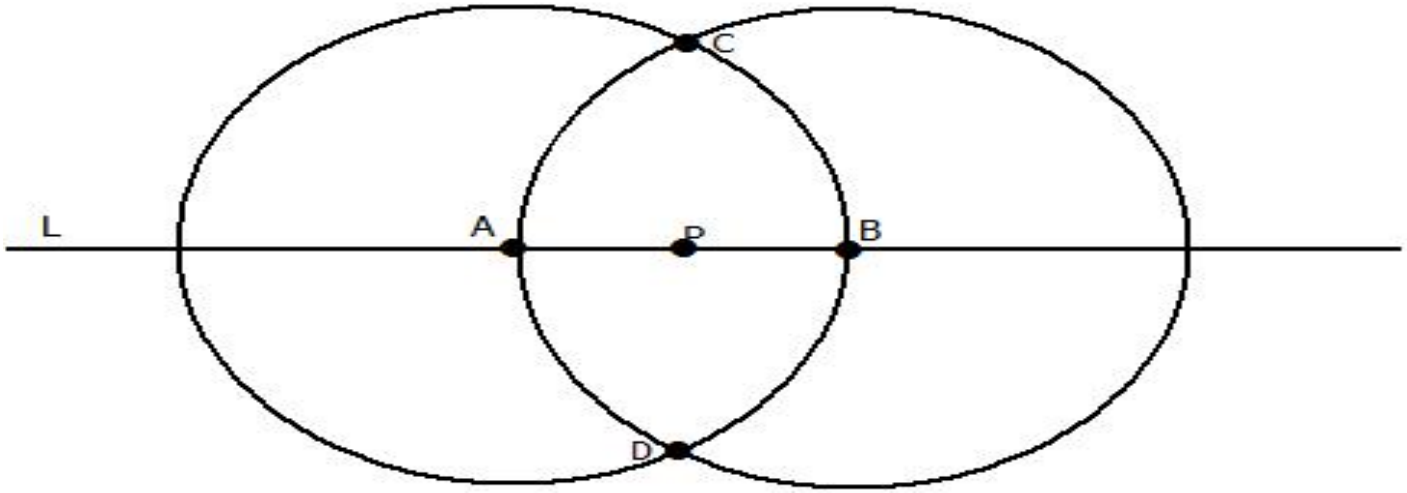
Examples of Basic Constructions



Theorem

Given a line L and a point P on the line, we can draw a line perpendicular to L that passes through P.

Examples of Basic Constructions



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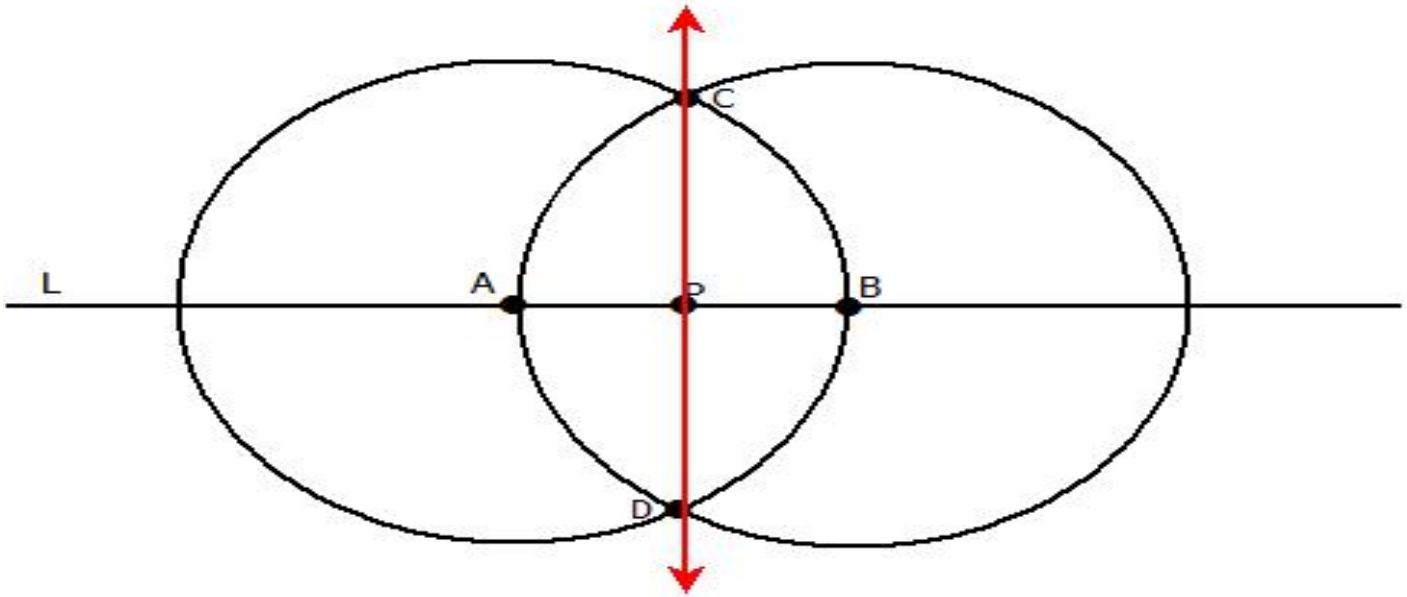

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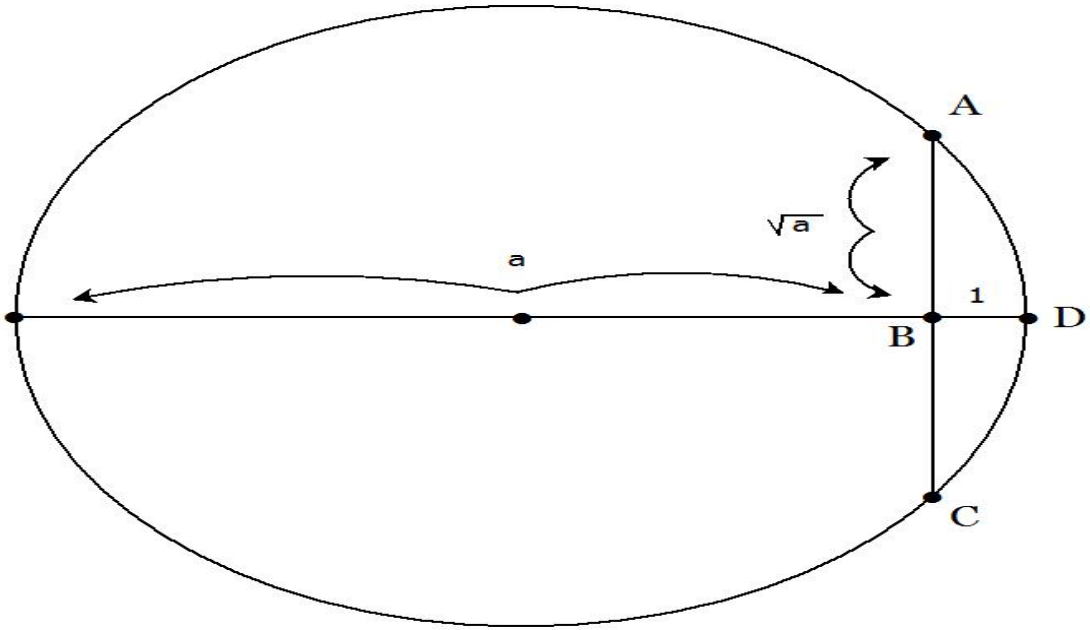
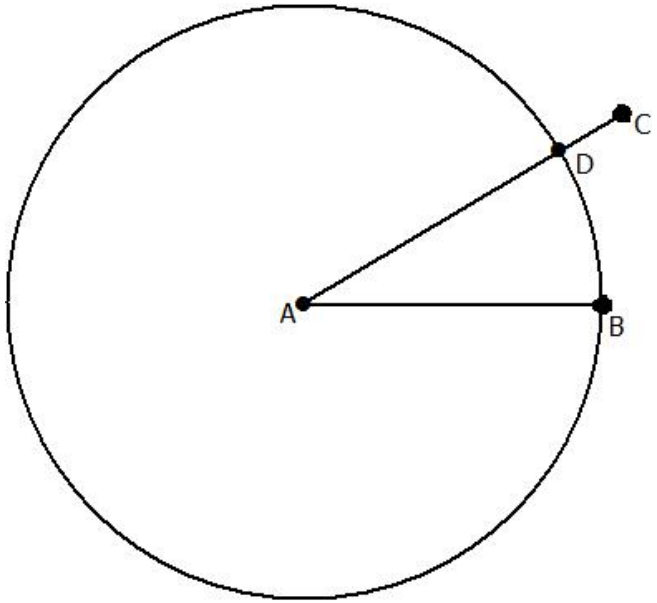


Theorem

Given a constructible number a, we can construct $\sqrt{a}$.

Examples of Basic Constructions



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Given an angle BAC, we can bisect the angle

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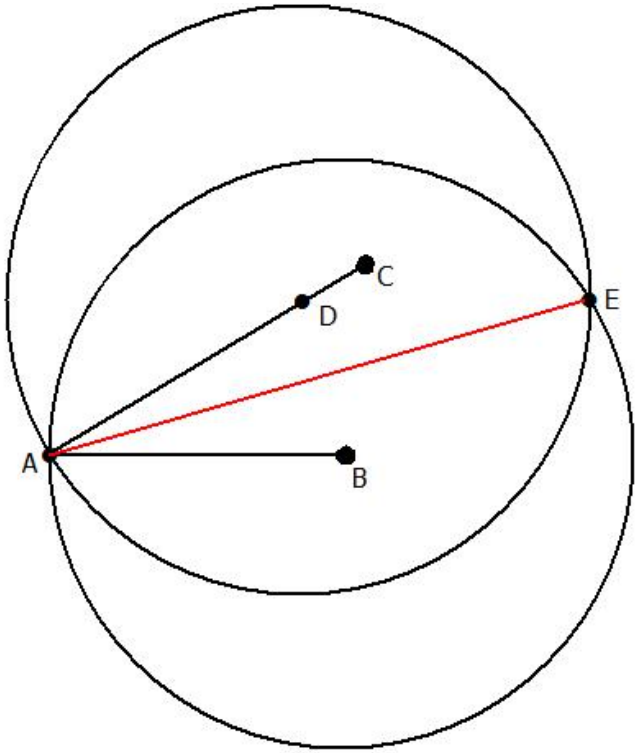

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Fields

- What is a field?

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- Operations

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  - Addition
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  - Multiplication
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- Inverses
  - Additive inverse
  - Multiplicative inverse

Examples of Fields

- $\mathbb{Q}$, the rational numbers

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Examples of Fields

- $\mathbb{Q}$, the rational numbers
- $\mathbb{R}$, the real numbers
- $\mathbb{C}$, the complex numbers
- $\mathbb{E}$, the constructible numbers

Two-tower over $\mathbb{Q}$ Theorem

Theorem

The number $\alpha \in \mathbb{R}$ is constructible with straight edge and compass ($\alpha \in$ constructiblenumbers) if and only there is a sequence of field extensions

$$\mathbb{Q} = F_0 \subset F_1 \subset F_2 \subset \ldots\ldots \subset F_n \text{ so that } [F_i : F_{i-1}] = 2 \text{ or } 1 \text{ for}$$

$$i = 1, \ldots\ldots, n \text{ (i.e. } F_i = F_{i-1}(\sqrt{\beta_i})), \beta_i \in F_{i-1} \text{ and } \alpha \in F_n$$

The Theorem states

$(\mathbb{Q})$ *is constructible with only a straight edge and compass.*

$\sqrt{\frac{p}{q}}$ *is constructible with only a straight edge and compass*

Limitations of Basic Constructions

What constructions are we not able to do with simply a straightedge and compass?

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- Taking the cube root of a constructible number

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Why is this?

Definitions and Rules for Neusis Construcitons

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- Postulates for Neusis Constructions

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  - Using a straightedge and compass we can:
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  - Using a twice notched straightedge and compass we can:
    - Pivot the twice notched straightedge around a point

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  - Using a twice notched straightedge and compass we can:
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  - Using the intersections of the slid and/or pivoted straightedge and those previously constructed lines and cirlces to constuct new points

Examples of Neusis Constructions

- Trisection of an angle

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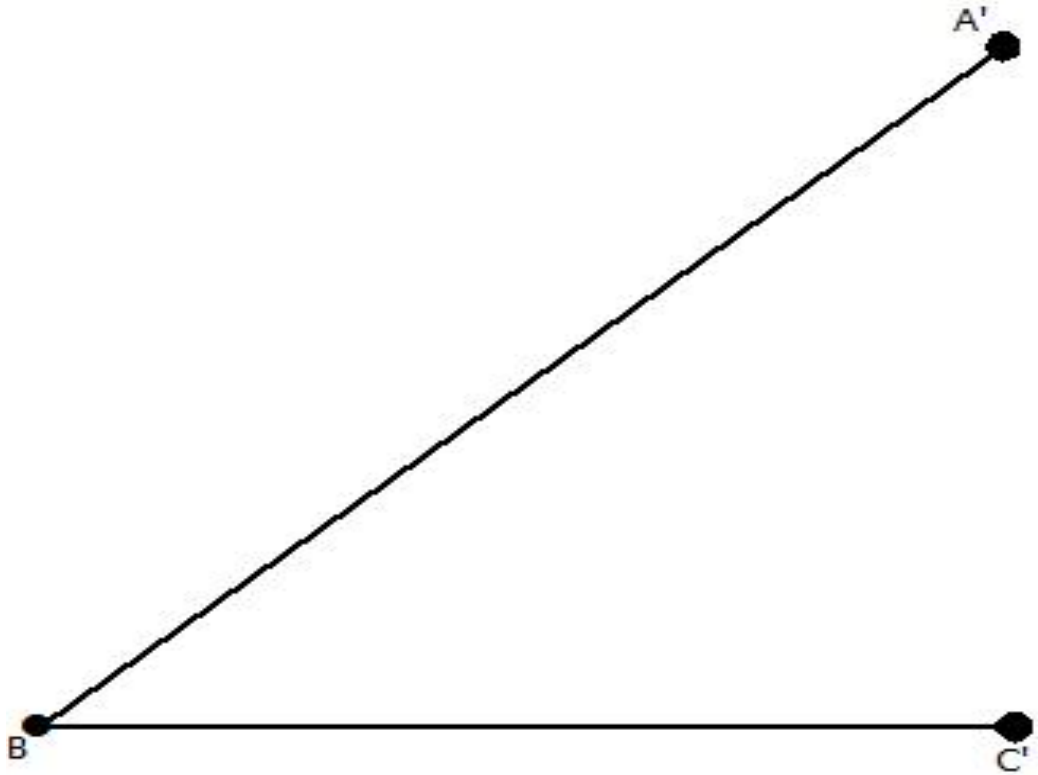
- Cube roots

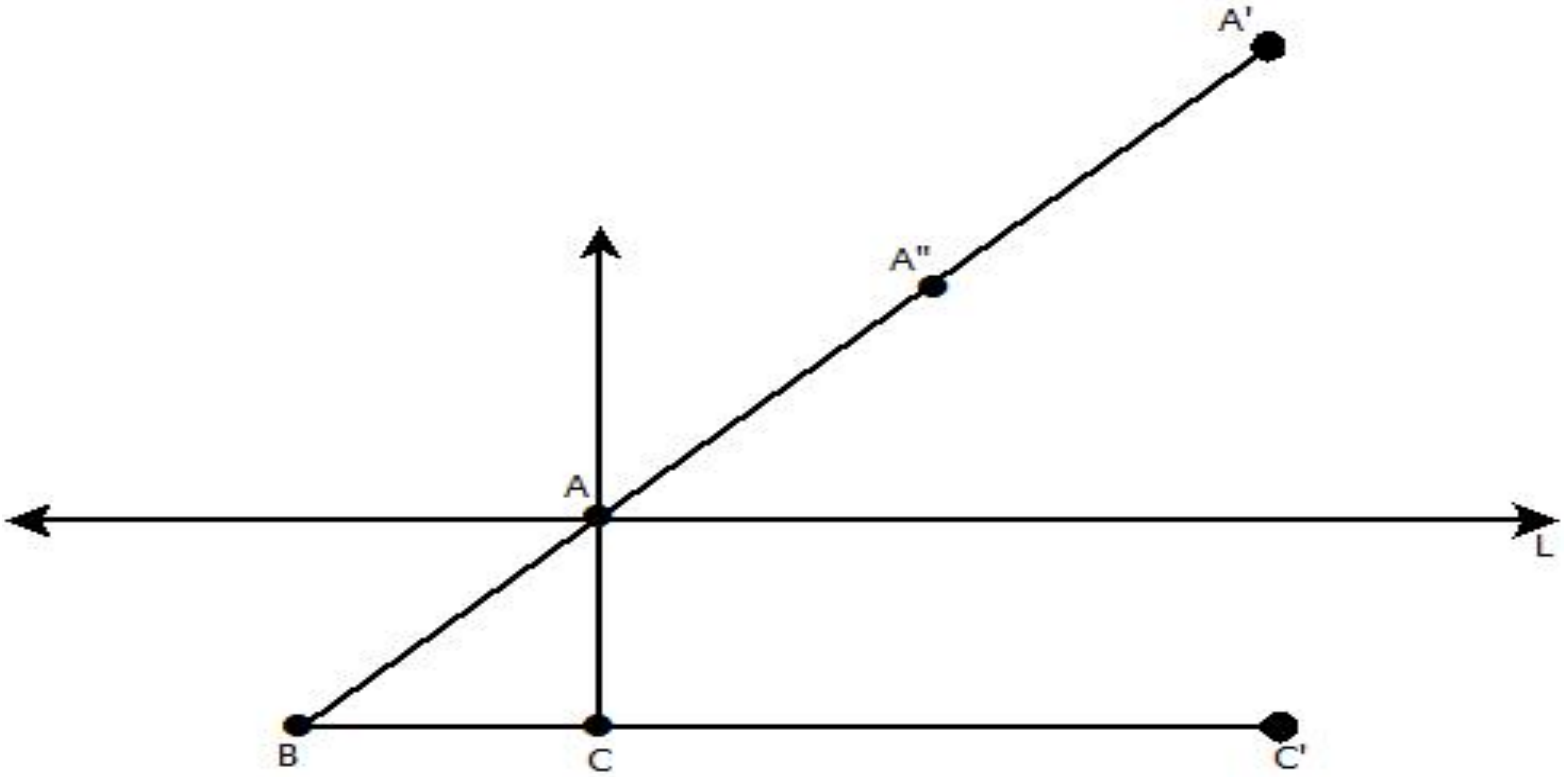
Neusis Constuction for trisection of an angle



Theorem

A triscetum construction using Neusis and given angle A'BC'

Neusis Constuction for trisection of an angle



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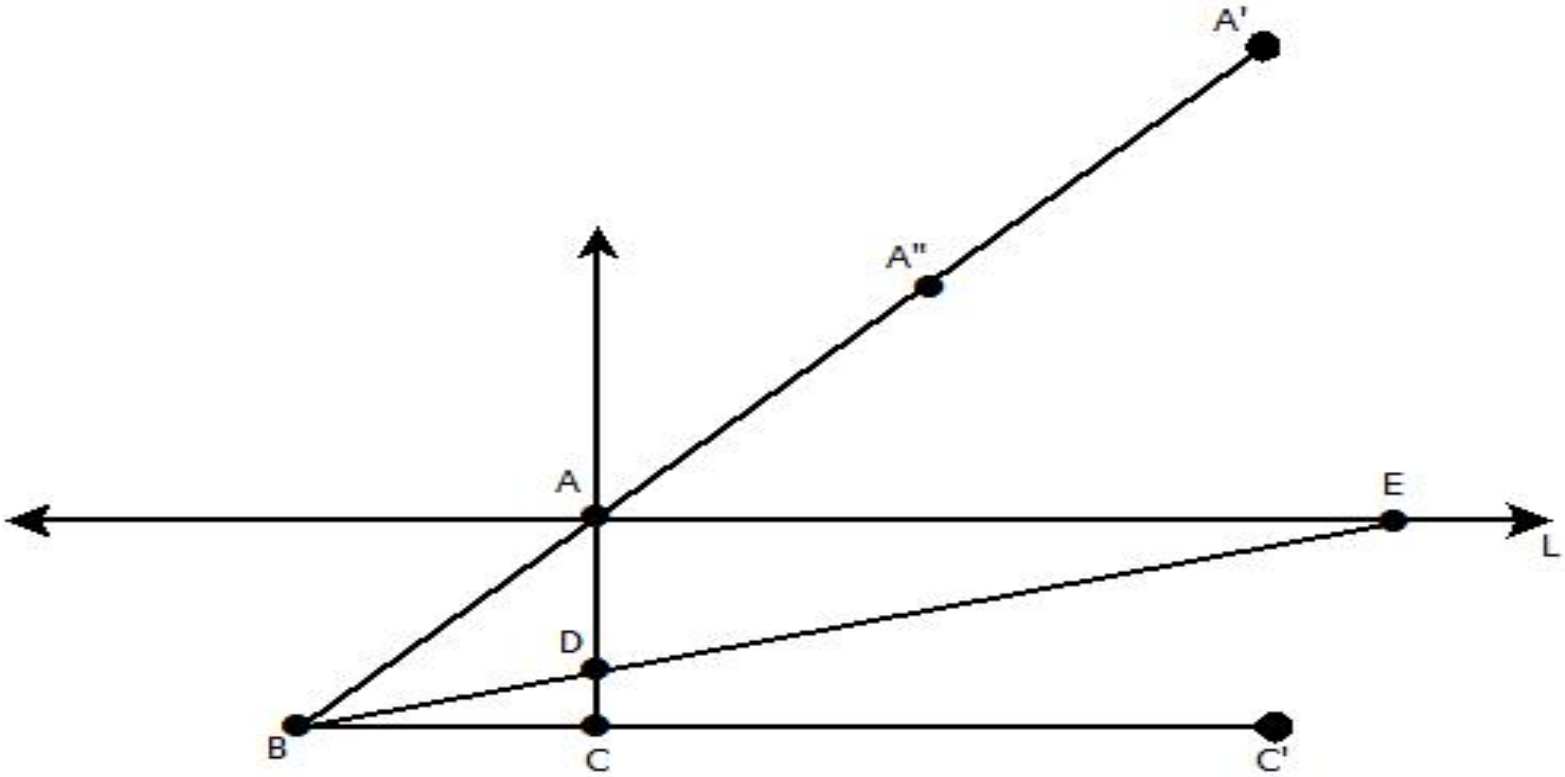

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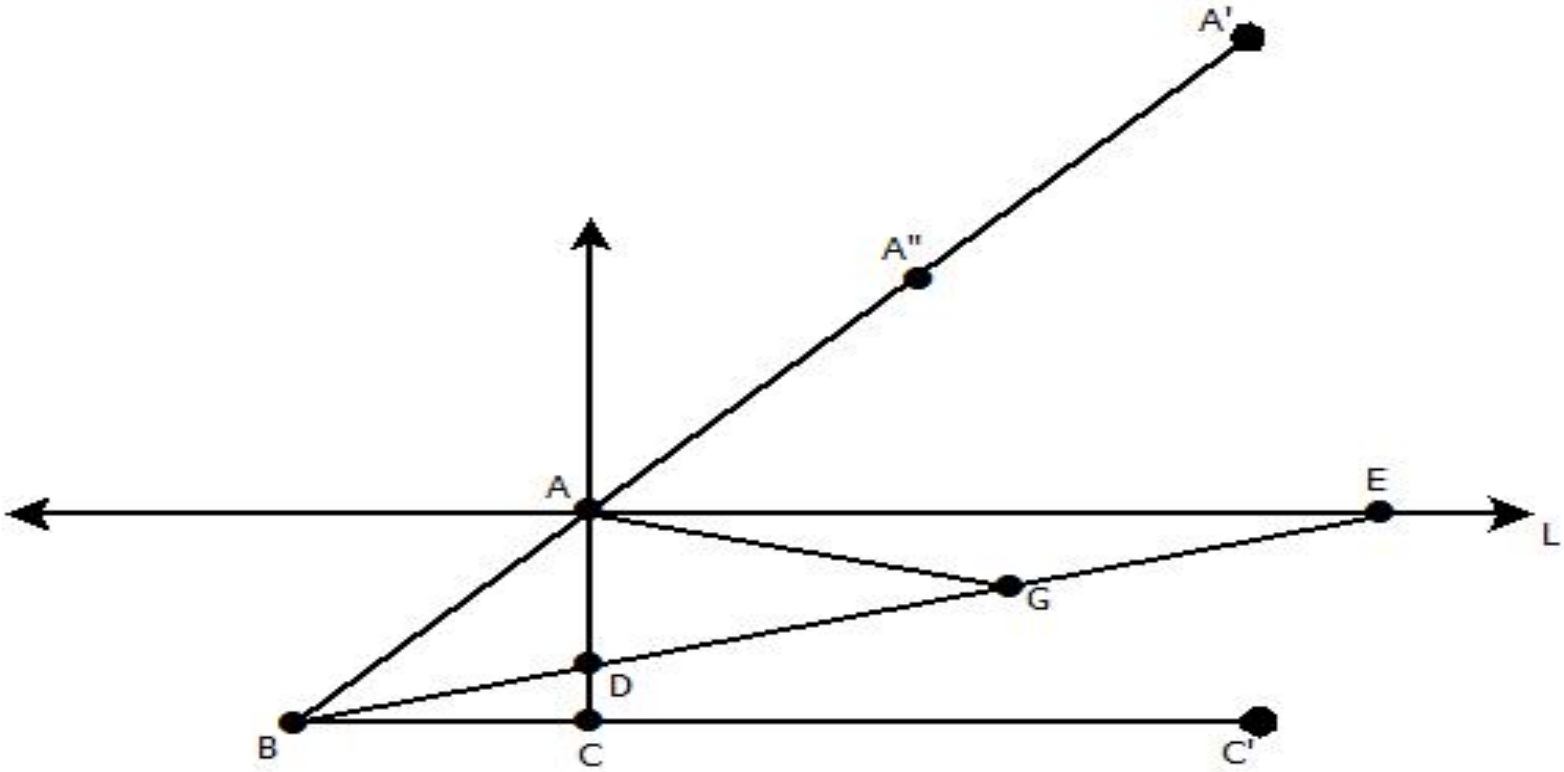

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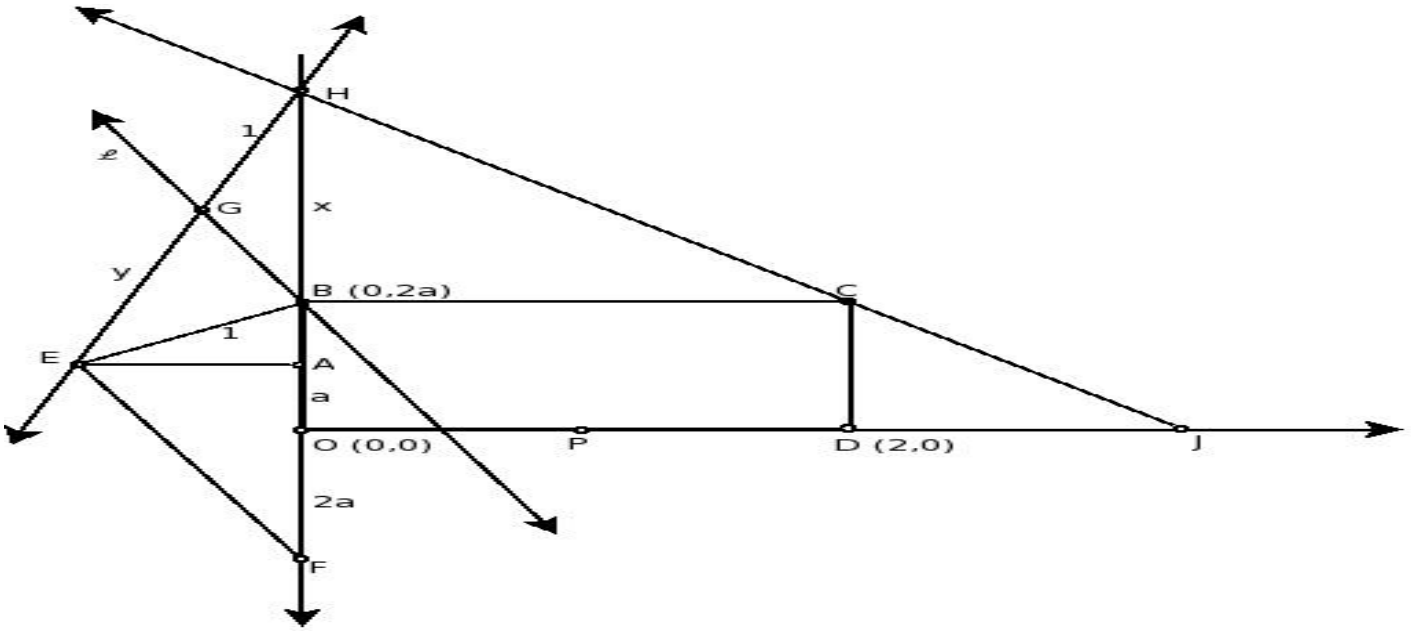
Neusis Constuction for cube roots



Theorem

Given a constructible length a, it is possible to find $\sqrt[3]{a}$ using a compass and twice-notched straightedge.

Claim: We can calculate $x = 2\sqrt[3]{a}$ by using similar triangles and proportions.

2-3 tower over $\mathbb{Q}$

Theorem

If a number $\alpha \in (\mathbb{R})$ is in F_n so that there is a sequence of field extensions

$$(\mathbb{Q}) = F_0 \subset F_1 \subset ..... \subset F_n \text{ with } [F_i : F_{i-1}] = 1, 2, 3$$

for $i = 1, ..., n$, then α is constructible with straight edge with two notches and compass (i.e. $\alpha \in$ the constructible numbers with two notches.)

Conclusion

- Basic Constructions versus Neusis Constructions

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- Fields and field extensions

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- Numbers we can construct

Conclusion

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- Is there a real number not degree $2^p 3^q$ over $\mathbb{Q}$ that can be constructed using a twice notched straight edge?

Acknowledgements

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