1. Projects in arithmetic algebraic geometry

Topics in this broad area have occurred in previous REU projects directed at LSU: (1) zeta functions of varieties and modularity (1999, 2005, 2008, 2010); (2) PF differential equations (2010, 2011). These subjects are accessible even to undergraduates in part because powerful software tools are available (e.g., SAGE, MAGMA) to their study and has led to journal publication [FHL$^+$10, HLV].

2. Recursions, differential equations and modular forms

Certain integer sequences defined by recursion relations have become important in several areas of number theory and algebraic geometry. The most famous are the Apéry sequences which occur in rational approximations to the zeta values $\zeta(2), \zeta(3)$, one of which is $a_0 = 1, a_1 = 5, 73, 1445, 33001, 819005, ...$ defined by

$$n^3 a_n + (n-1)^3 a_{n-2} = (34n^3 - 51n^2 + 27n - 5)a_{n-1}.$$

It was discovered by Beukers that if you define $a(t) = \sum a_n t^n$ then this function satisfies the differential equation

$$t(t^2 + 11t - 1)a''(t) + (3t^2 + 22t - 1)a'(t) + (t + 3)a(t) = 0.$$

He also discovered that this DE is the Picard-Fuchs differential equation satisfied by the periods of the elliptic curves E_t in the family

$$E_t: \quad y^2 = x^3 + \frac{1}{4}(1 + 6t + t^2)x^2 + \frac{1}{2}t(t+1)x + \frac{1}{4}t^2.$$

Later Beukers, Stienstra, Peters, Zagier and others investigated families of differential equations coming from algebraic geometry for which there was a holomoprhic solution to that equation with *integer* expansion coefficients. It was also shown that these integer sequences satisfied congruences modulo prime powers (Atkin-Swinnerton-Dyer or Cartier-Honda congruences). [Beu83], [BP84], [FHL$^+$10], [HLV], [Kat73], [Kib], [Moy], [Pet86], [SB85], [Sti87a], [Sti87b], [Ver01], [Ver10], [Ver96], [Zag09]

In summer 2010 and 2011 the LSU REU investigated new examples of this phenomenon.

Polytopes, toric geometry, GKZ systems, Calabi-Yau varieties

Here the idea is to consider families of Calabi-Yau varieties that arise from toric geometry. Toric geometry is a branch of algebraic geometry that relates the theory of lattices and polytopes to the construction of algebraic goemetry. One main attraction of this subject is that it involves elementary and concrete ideas, accessible to undergraduates, yet it connects to many deep and active areas of mathematical research. In particular, toric geometry affords some constructions of Calabi-Yau varieties. These CY varieties have also been the focus of intense interest, not only because of their connection to physics, but also because of their rich geometric and arithmetic properties. Indeed, even physicists have made nontrivial discoveries in the arithmetic of CY varieties.

Many of the researches around CY varieties involve explicit computation with and study of examples. These examples are in some cases generalizations of examples that have already been explored in the LSU REU in previous years. For instancs, in 2010 we studied the symmetric squares of Picard Fuchs differential equations of families of elliptic curves; these are the PF equations of a certain K3-fibration of a rigid Calabi-Yau threefold. Our proposal here is to go further in this direction, to investigate Calabi-Yau varieties, especially those amenable to concrete and explict realization. We can investigate such issues as

(1) Calculation of the zeta function.
(2) Modularity (automorphy) of the related Galois representations.
(3) Picard-Fuchs differential equations.

References: [CLS11], [CdlORV03], [CdlO07], [GKZ08], [HV05], [Sti07], [TY07], [Yui00], [Yui03]

Hilbert modular forms

We propose to also compute examples involving Hilbert modular forms. Already these forms have appeared in modularity questions concerning Calabi-Yau varieties. Also, some computed examples involving non-congruence modular forms conjecturally are related to Hilbert modular forms. References: [DV], [Gru10], [HM95], [KV07], [Voi].

Projects in Constructive Galois Theory

This area of mathematics is particularly well-suited for undergraduate research. There are many entry points to the theory that are natural, concrete and computationally explicit.

Let K be a field and let G be a finite group. A polynomial $f \in K(t_1, \ldots, t_n)[X]$ is said to be *generic* for G over K if it satisfies the following conditions: (i) The Galois group of f over $K(t_1, \ldots, t_n)$ is isomorphic to G, and (ii) any Galois extension M/L with group G, where L is a field containing K, can be obtained by suitably specializing $t_1, \ldots, t_n$ in L. For example, the polynomial $X^3 - tX^2 + (t-3)X + 1$ is generic for the cyclic group C_3 over any field and $X^4 - 2t_1t_2X^2 + t_1^2t_2(t_2-1)$ is generic for the dihedral group D_4 over a field of characteristic different from 2 (see [JLY02, Ch. 2]).

If the ground field K is *Hilbertian* (e.g. number fields), the existence of a G-generic polynomial over K implies in particular that G is realizable as Galois group over K.

The research projects in this area will focus on questions of existence and computation of generic polynomials for some groups and families of groups. Explicit computations are more feasible today than ever with the powerful software tools for group theory, invariant theory and commutative algebra that are available in software packages such as SAGE, GAP, MAGMA. Our projects will also address subsidiary questions such as the minimal number of parameters needed (the *generic dimension*) and its relation with the notion of *essential dimension* introduced by Buhler and Reichstein [BR97].

The question of the existence of generic polynomials for a given group is nontrivial. There are very few general results.

We summarize below some of the techniques used to compute generic polynomials.

Invariant theory

The classical Noether problem asks whether the invariant field of a subgroup $G \subset S_n$ acting on the field of rational functions $K(x_1, \ldots, x_n)$ by permutation of the indeterminates is purely transcendental over K. If the answer is affirmative, then then the polynomial $F = (X - x_1) \cdots (X - x_n)$ expressed in terms of a transcendence basis of $K(x_1, \ldots, x_n)^G$ parametrizes all polynomials with Galois group G. It is in particular a generic polynomial in the above sense.

Kemper [KM00] showed that if G is a finite subgroup of $GL_n(K)$ that acts on $K(x_1, \ldots, x_n)$ by linear transformation of the indeterminates and such that $K(x_1, \ldots, x_n)^G$ is purely transcendental over K, then there is an explicit procedure to construct a generic G-polynomial.

Frobenius Modules

Let G be a connected linear algebraic group defined over $\mathbb{F}_q$. Matzat's theory of Frobenius modules [Mat04] allows the construction of polynomials with coefficients in $\mathbb{F}_q(t_1, \ldots, t_n)$ with Galois group $G(\mathbb{F}_q)$. For instance, all the classical groups of Lie type haven realized as Galois groups over such fields by Albert and Meier [AM09] using these techniques. It is not clear at all that these polynomials are generic. It seems that by starting out with a "sufficiently general" (notion to be made precise) Frobenius module, one should be able to produce generic polynomials.

We discuss below some examples of projects.

(1) *Frobenius modules and generic polynomials.* This project will study the construction of generic polynomials in characteristic p using Matzat's Frobenius module approach [Mat04]. Explicit polynomials with coefficients in $\mathbb{F}_q(t_1, \ldots, t_n)$ have recently been obtained for essentially all the simple groups of Lie type by M. Albert and A. Maier [AM09] using this method. It is unclear whether these polynomials are generic. One interesting particular case is that of orthogonal groups. It would be interesting to settle this question even for small cases.

(2) *Generic polynomials for p-groups in characteristic p.* Let $U \subset \mathrm{GL}_n$ be a closed, connected unipotent algebraic subgroup defined over $\mathbb{F}_q$. The Frobenius module approach should produce generic polynomials for $U(\mathbb{F}_q)$ in m parameters, where $m = \dim(U)$ (note also that $|U(\mathbb{F}_p)| = p^m$). A related question is whether every p-group is of the form $U(\mathbb{F}_p)$ for some *connected* unipotent algebraic group U defined over $\mathbb{F}_p$. This is the case for abelian p-groups by the theory of Witt vectors, but seems to be unknown in general.

3. Projects in Combinatorial Invariants in Classical Link Theory

Recent ribbon graph invariants have been prominent in the research study of low dimension knots and links. To each state smoothing of a link diagram there is an associated ribbon graph whose combinatorial properties invariants under Reidemeister equivalence of diagrams yield link invariants. Polynomial invariants of ribbon graph have been related to both the Jones [CP07, DFK$^+$08] (by a specialization of the Bollobas-Riordan-Whitney-Tutte polynomial [BR02]) and to the Alexander polynomial [Kau83, CDR10] (via the Kasteleyn determinant). Invariants of ribbon graphs and their properties are in their infancy and there are still great opportunities for advanced undergraduates to make an impact.

Given a hint concerning a good place to look undergraduates can learn the fundamentals and focus on a new direction simultaneously. Subsequently, the visual connections between the geometry of a link diagram and the state sums for Kauffman bracket/Jones polynomial help enhance the understanding and expertise by providing two distinct optics from which to *view* examples and problems.

$$R(\mathbb{D}_n) = \sum_{\mathbb{H} \subset \mathbb{D}_n} x^{k(\mathbb{H})-k(\mathbb{D}_n)} y^{n(\mathbb{H})} z^{g(\mathbb{H})}.$$

In fact, recent projects in this REU has already extended the transfer method of Biggs and his school from the Tutte polynomial of graphs to the BRWT of ribbon graphs.

Given an appropriate definition of constructible family of ribbon graphs, $\mathbb{D}_n$, as a ribbon graph amalgamation $\mathbb{D}_{n-1} \cup \mathbb{M}$ with a fixed pattern $\mathbb{M}$, the transfer methods yields a decomposition:

$$R(\mathbb{D}_n) = \sum_{\mathbb{H}_{n-1} = \mathbb{H} \cap \mathbb{D}_{n-1}} x^{k(\mathbb{H}_{n-1})-k(\mathbb{D}_n)} y^{n(\mathbb{H}_{n-1})} z^{g(\mathbb{H}_{n-1})} \sum_{\mathbb{M}_0 \subset \mathbb{M}} x^{\Delta k(\mathbb{H}_{n-1},\mathbb{M}_0)} y^{\Delta n(\mathbb{H}_{n-1},\mathbb{M}_0)} z^{\Delta g(\mathbb{H}_{n-1},\mathbb{M}_0)}$$

for $\mathbb{M}_0$ a spanning subgraph of $\mathbb{M}$.

The transfer method can be applied whether the second sum can be decomposed into a finite number of state transitions independent of the stage of the tower.

Recursion properties of Families of Graphs and Dessins.

(1) Extending the transfer method for Tutte pioneered by the school of Norman Biggs [BDS72] , find recursions for BRWT for interesting families of graphs, including families formed by iterated partial twisting[CK05] and extends the results formulated by Jablan, Radović,and Sazdanović [JRS10] for alternating link diagrams (the case of ribbon graphs with genus zero) and the Tutte polynomial.
(2) An infinite family is constructible from the ribbon graph of the quotient of an link diagram invariant under a cyclic group action.

Medial graph construction from rank two and higher GKZ systems. Given a rank two integer matrix in $\mathbb{Z}^N$ from a GKZ system, Jan Stienstra [Sti08] constructs a ribbon graph on a torus, the zig-zag link (Figure 11) as well as a related Kasteleyn multi-variable polynomial. Undergraduates can explore these examples by constructing a program to compute the dessin of the zig-zag of the above construction then explore the specializations of Kasteleyn bi-adjacency matrix of the zig-zag which are link invariants. The Alexander polynomial is one such invariant via the work of Dasbach, Russell and Cohen *arXiv:1010.5228*, in their Kasteleyn re-interpretation of an approach of Kauffman in his book *Formal Knot Theory.*

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