Section 10.2 Inverse Functions

# Objective 1: Determining Whether a Function is One-to-One

Before we begin looking at inverse functions, we first need to define one-to-one functions. We know that in order for a relation to be a function, each input must correspond to exactly one output. When a function is such that each output corresponds to exactly one input, then the function is called a **one-to-one function**.

**One-to-One Function:**

For a one-to-one function, each -value (input) corresponds to only one -value (output), and each -value (output) corresponds to only one -value (input).

Is a one-to-one function? Explain your reasoning.

# Objective 2: Using the Horizontal Line Test

One way to quickly determine if a function is a one-to-one function is to look at its graph and apply the **horizontal line test**.

**Horizontal Line Test:**

If every horizontal line intersects the graph of a function at most once, then the function is a one-to-one function.

Determine if the function is a one-to-one function by sketching its graph.

|  |  |
| --- | --- |
| a. | b. |

|  |  |
| --- | --- |
| c. | d. |

# Objective 3: The Inverse of a Function

For each one-to-one function, we can find its **inverse function**. If a function is not one-to-one, then it does not have an inverse function.

Inverse functions “undo” each other. If you take the output of one function for a given input and put the output into the other function, you get the original input back.

For a function , we use the notation , read “ inverse,” to denote its inverse function.

**Inverse Function:**

The inverse of a one-to-one function is the one-to-one function that consists of the set of all ordered pairs where belongs to .

Note that since the coordinates of each ordered pair have been switched, the domain of is the range of , and the range of is the domain of .

a. Consider the function and its inverse . Complete the tables.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

b. Consider a one-to-one function . Fill in the blanks. If , then .

# Objective 4: Finding the Equation of an Inverse Function

Given the equation of a one-to-one function , we can find the equation of by interchanging and in the equation of and then solving for .

For example, given the equation , the equation of can be found as follows:

The equation of the inverse function is .

Find the equation of . Graph and on the same axes.

|  |  |
| --- | --- |
| a. | b. |
| Blank coordinate plane that spans from negative ten to positive ten on each axis with a scale of one unit. | Blank coordinate plane that spans from negative ten to positive ten on each axis with a scale of one unit. |

# Objective 5: Graphing Inverse Functions

Notice from the graphs in the previous section that the graphs of and are mirror images of each other. They are reflections of each other about the line . This is true for every function and its inverse.

Find the equation of the inverse function for . Graph and on the same axes.

