Section 10.2 Inverse Functions

# Objective 1: Determining Whether a Function is One-to-One

Before we begin looking at inverse functions, we first need to define one-to-one functions. We know that in order for a relation to be a function, each input must correspond to exactly one output. When a function is such that each output corresponds to exactly one input, then the function is called a **one-to-one function**.

**One-to-One Function:**

For a one-to-one function, each $x$-value (input) corresponds to only one $y$-value (output), and each $y$-value (output) corresponds to only one $x$-value (input).

Is $f\left(x\right)=x^{2}$ a one-to-one function? Explain your reasoning.

# Objective 2: Using the Horizontal Line Test

One way to quickly determine if a function is a one-to-one function is to look at its graph and apply the **horizontal line test**.

**Horizontal Line Test:**

If every horizontal line intersects the graph of a function at most once, then the function is a one-to-one function.

Determine if the function is a one-to-one function by sketching its graph.

|  |  |
| --- | --- |
| a. $f\left(x\right)=|x-5|$ | b. $f\left(x\right)=x^{3}+4$ |

|  |  |
| --- | --- |
| c. $f\left(x\right)=\frac{2}{3}x+5$ | d. $f\left(x\right)=-3$ |

# Objective 3: The Inverse of a Function

For each one-to-one function, we can find its **inverse function**. If a function is not one-to-one, then it does not have an inverse function.

Inverse functions “undo” each other. If you take the output of one function for a given input and put the output into the other function, you get the original input back.

For a function $f$, we use the notation $f^{-1}$, read “$f$ inverse,” to denote its inverse function.

**Inverse Function:**

The inverse of a one-to-one function $f$ is the one-to-one function $f^{-1}$ that consists of the set of all ordered pairs $(y,x)$ where $(x,y)$ belongs to $f$.

Note that since the coordinates of each ordered pair have been switched, the domain of $f$ is the range of $f^{-1}$, and the range of $f$ is the domain of $f^{-1}$.

a. Consider the function $f\left(x\right)=x+4$ and its inverse $f^{-1}\left(x\right)=x-4$. Complete the tables.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ | $$f(x)$$ |  | $$x$$ | $$f^{-1}(x)$$ |
| $$-3$$ |  |  | $$1$$ |  |
| $$0$$ |  |  | $$4$$ |  |
| $$5$$ |  |  | $$9$$ |  |

b. Consider a one-to-one function $f$. Fill in the blanks. If $f\left(3\right)=12$, then $f^{-1}(\\_\\_\\_\\_\\_)=\\_\\_\\_\\_\\_$.

# Objective 4: Finding the Equation of an Inverse Function

Given the equation of a one-to-one function $f$, we can find the equation of $f^{-1}$ by interchanging $x$ and $y$ in the equation of $f$ and then solving for $y$.

For example, given the equation $f\left(x\right)=x+4$, the equation of $f^{-1}$ can be found as follows:

$$x=y+4$$

$$x-4=y$$

The equation of the inverse function is $f^{-1}\left(x\right)=x-4$.

Find the equation of $f^{-1}$. Graph $f$ and $f^{-1}$ on the same axes.

|  |  |
| --- | --- |
| a.$ f\left(x\right)=2x+4$ | b. $f\left(x\right)=-\frac{1}{4}x$ |
| Blank coordinate plane that spans from negative ten to positive ten on each axis with a scale of one unit. | Blank coordinate plane that spans from negative ten to positive ten on each axis with a scale of one unit. |

# Objective 5: Graphing Inverse Functions

Notice from the graphs in the previous section that the graphs of $f$ and $f^{-1}$ are mirror images of each other. They are reflections of each other about the line $y=x$. This is true for every function and its inverse.

Find the equation of the inverse function for $f\left(x\right)=x^{3}-4$. Graph $f$ and $f^{-1}$ on the same axes.

