Section 10.5 Properties of Logarithms

# Objective 1: Using the Product Property

The first property we will explore is called the **product property of logarithms** because it deals with the logarithm of a product.

**Product Property of Logarithms:**

If , , and are positive real numbers and , then

# This property can be proven using a property of exponents. Let and

Then, is equivalent to , and is equivalent to .

Rewriting in logarithmic form, is equivalent to

Therefore,

This property can be used to expand or condense logarithmic expressions.

# Write as a single logarithm. Assume that variables represent values for which each logarithm is defined.

|  |  |
| --- | --- |
| a.  | b.  |

# Write as a sum of logarithms. Evaluate when possible. Assume that variables represent values for which each logarithm is defined.

|  |  |
| --- | --- |
| c.  | d.  |

# Objective 2: Using the Quotient Property

The second property deals with the logarithm of a quotient and is therefore called the **quotient property of logarithms**.

**Quotient Property of Logarithms:**

If , , and are positive real numbers and , then

a. Use a property of exponents to prove the quotient property of logarithms.

# As with the product property, the quotient property of logarithms can be used to expand or condense logarithmic expressions.

# Write as a single logarithm. Assume that variables represent values for which each logarithm is defined.

|  |  |
| --- | --- |
| b.  | c.  |

# Write as a difference of logarithms. Evaluate when possible. Assume that variables represent values for which each logarithm is defined.

|  |  |
| --- | --- |
| d.  | e.  |

# Objective 3: Using the Power Property

The third property that we will explore in this section is the **power property of logarithms**. It deals with the logarithm of an expression raised to an exponent.

**Power Property of Logarithms:**

If and are positive real numbers and and is a real number, then

This property can be proven by using what we know about logarithms.

Let . Then, . Using substitution,

The expression evaluates to equal . So

.

Thus, .

# Use the power property to rewrite the expression so that the logarithm does not contain an exponent.

|  |  |  |
| --- | --- | --- |
| a.  | b.  | c.  |

# Use the power property to rewrite the expression so that the coefficient of the logarithm is .

|  |  |
| --- | --- |
| d.  | e.  |

# Objective 4: Using the Properties Together

We often need to use some combination of the three properties to expand or condense logarithmic expressions.

# Write as a single logarithm. Assume that variables represent values for which each logarithm is defined.

|  |  |
| --- | --- |
| a.  | b.  |
| c.  | d.  |

# Rewrite the expression so that no logarithm contains a product, quotient, or exponent. Evaluate when possible. Assume that variables represent values for which each logarithm is defined.

|  |  |
| --- | --- |
| e.  | f.  |

|  |  |
| --- | --- |
| g.  | h.  |
| i.  |  |

Given that and , use the properties of logarithms to evaluate.

|  |  |
| --- | --- |
| j.  | k.  |

|  |  |
| --- | --- |
| l.  | m.  |