Section 14.5 Graphs of Sine and Cosine Functions

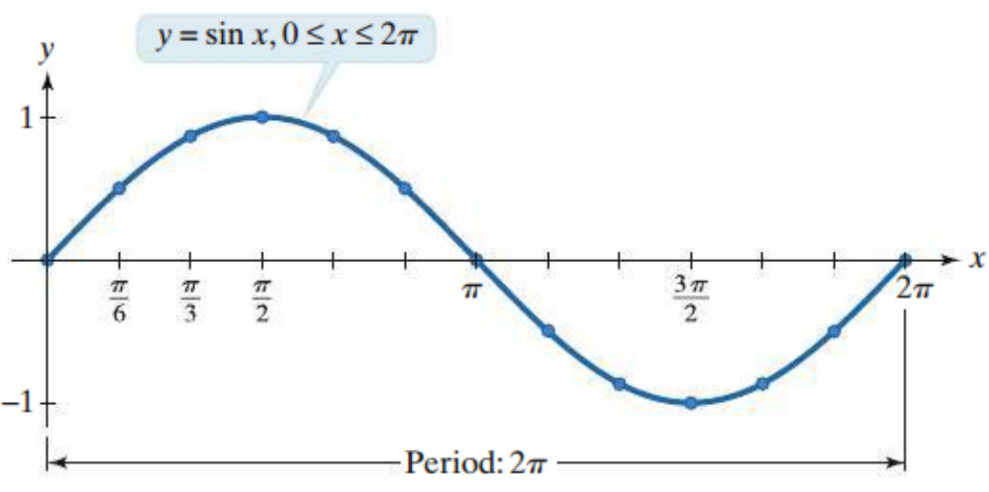
# Objective 1: The Graph of

We can graph by finding and plotting select points , where the *x*-value represents an angle measured in radians and the *y*-value represents the sine of the angle.

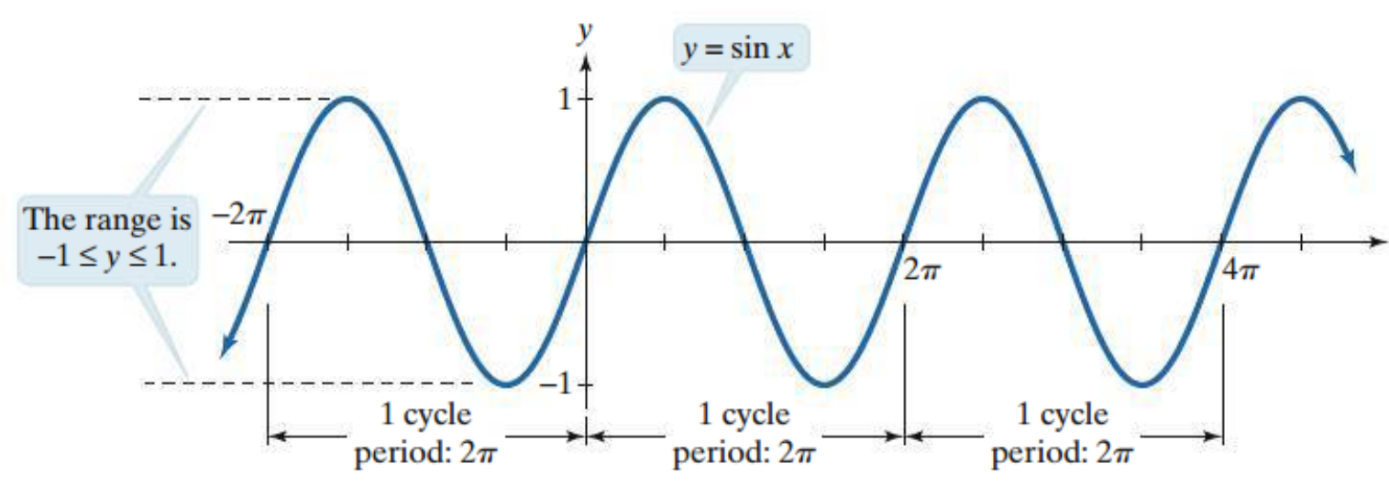
The table shows some select values that we can use to graph the function. Because the period of the sine function is , we will graph the function on the interval . The rest of the graph is made up of repetitions of this portion.

A table two rows. Row 1 is x 0, pi/6, pi/3, pi/2, 2pi/3, 5pi/6, pi, 7pi/6, 4pi/3, 3pi/2, 5pi/3, 11pi/6, 2pi. Row 2 is y = sinx and the value of sine for each x-value.

Below the table is a description of the value of sine on the intervals 0 to pi/2, pi/2 to pi, pi to 3pi/2, and 3pi/2 to 2pi.



We can obtain a more complete graph of the sine function by graphing more cycles. Any part of the graph that corresponds to one period () is one cycle of the graph of .

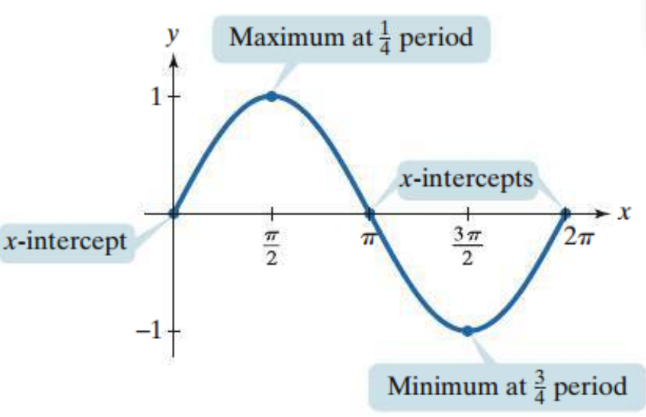


**Characteristics of the Sine Function and Its Graph:**

* The domain is all real numbers.
* The range is .
* The period is .
* The function is odd. The graph is symmetric about the origin.
* The -intercepts of the graph are of the form where is an integer.

# Objective 2: Graphing Variations of

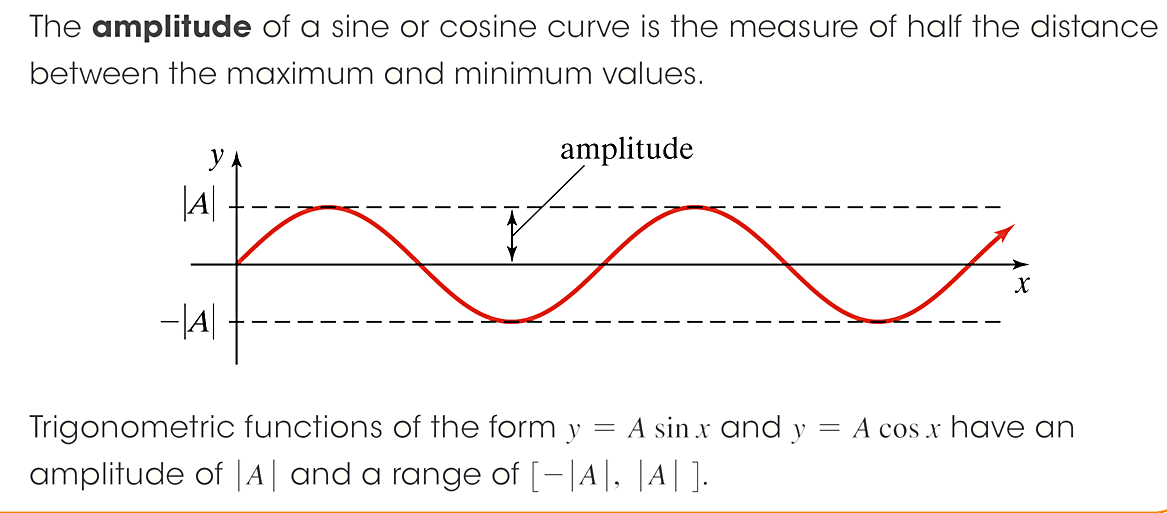
To graph one complete cycle of any transformation of the sine function, it is necessary to find a minimum of five points: three -intercept points, one maximum point, and one minimum point. Notice that on the graph of , these five points evenly divide one cycle into fourths or quarters. The -coordinates of the five key points can be defined as follows.



value of where the cycle begins

The -coordinates of the five key points are obtained by evaluating the given function at each of these -values.

The **amplitude** of a sine curve is the measure of half the distance between the maximum and minimum values.



Trigonometric functions of the form have an amplitude of and a range of .

Determine the amplitude of the function. Then graph the function and on the interval on the same coordinate plane.

|  |  |
| --- | --- |
| a. | b. |
| c. |  |

We will now look at functions of the form where . Because the graph of completes one cycle from to , we know that the graph of completes one cycle as increases from to .

So the graph of completes one full cycle from to . The period is .

**Amplitude and Period:**

The graph of has amplitude = and period = .

Determine the amplitude and period of the function. Then graph one period of the function.

|  |  |
| --- | --- |
| d. | e. |
| f. | g. |

We will now consider functions of the form where . The graph still has amplitude = and period = . The graph completes one cycle as increases from to .

is the value of the -coordinate on the left where the cycle begins, and is the value of the -coordinate on the right where the cycle ends. The graph of is the graph of shifted horizontally units. This horizontal shift of is called the **phase shift** of the graph of the sine function.

Determine the amplitude, period, and phase shift of the function. Then graph one period of the function.

|  |  |
| --- | --- |
| h. | i. |
| j. | k. |

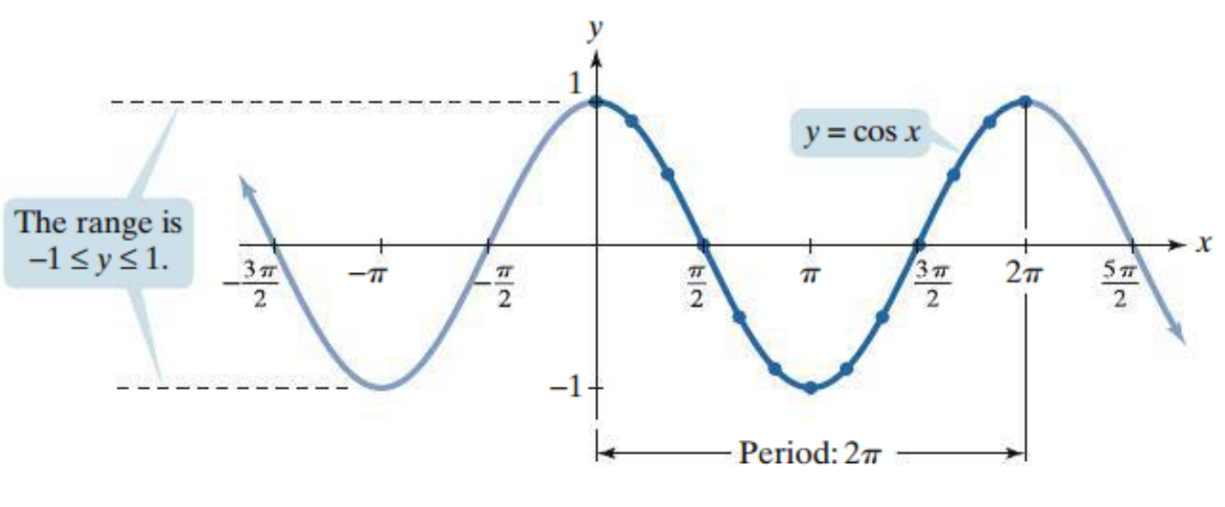
# Objective 3: The Graph of

As we did to first graph the sine function, we can graph by finding and plotting select points , where the *x*-value represents an angle measured in radians and the *y*-value represents the cosine of the angle.

The table shows some select values that we can use to graph the cosine function.

A table two rows. Row 1 is x 0, pi/6, pi/3, pi/2, 2pi/3, 5pi/6, pi, 7pi/6, 4pi/3, 3pi/2, 5pi/3, 11pi/6, 2pi. Row 2 is y = cosx and the value of cosine for each x-value.

Below the table is a description of the value of cosine on the intervals 0 to pi/2, pi/2 to pi, pi to 3pi/2, and 3pi/2 to 2pi.



**Characteristics of the Cosine Function and Its Graph:**

* The domain is all real numbers.
* The range is .
* The period is .
* The function is even. The graph is symmetric about the -axis.
* The -intercepts of the graph are of the form where is an odd integer.

# Objective 4: Graphing Variations of

We will graph variations of the graph of the cosine function in the same way that we graphed variations of the sine function.

The graph of has amplitude = and period = . The graph of is the graph of shifted horizontally units. In other words, the graph of has a phase shift of .

Determine the amplitude and period. Then graph one period of the function.

|  |  |
| --- | --- |
| a. | b. |

Determine the amplitude, period, and phase shift of the function. Then graph one period of the function.

|  |  |
| --- | --- |
| c. | d. |
| e. | f. |

# Objective 5: Vertical shifts of Graphs of Sine and Cosine Functions

We will now look at graphs of and . The constant shifts the graph of and vertically units.

Graph one period of the function.

|  |  |
| --- | --- |
| a. | b. |
| c. | d. |

**Objective 6: Modeling Periodic Behavior**

Many real-world situations are cyclical and can therefore be modeled with a sine or cosine function.

The number of hours of daylight on a given day in a certain city is given by the equation where represents the number of days after January .

a. How many hours of daylight are there on the longest day of the year?

b. How many hours of daylight are there on the shortest day of the year?