Section 7.1 Rational Expressions

# Objective 1: Finding the Domain of a Rational Expression

A **rational expression** is an expression that can be written as the quotient $\frac{P}{Q}$ of two polynomials $P$ and $Q$ as long as $Q$ is not $0$. The following are some examples of rational expressions.

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| --- | --- | --- |
| $$\frac{3x+7}{2}$$ | $$\frac{5x^{2}-3}{x-1}$$ | $$\frac{7x-2}{2x^{2}+7x+6}$$ |

Rational expressions are sometimes used to describe functions. For example, the function $f\left(x\right)=\frac{5x^{2}-3}{x-1}$ is a **rational function** since $f$ is defined by a rational expression.

A rational expression is undefined if the denominator is $0$. For example, the expression $\frac{5x^{2}-3}{x-1}$ is undefined when $x$ is $1$. For this reason, we must exclude $1$ from the domain of the function $f\left(x\right)=\frac{5x^{2}-3}{x-1}$. The domain of $f$ is all real numbers except $1$. We can write the domain of $f$ using set builder notation as $\{x|x is a real number and x\ne 1\}$.

Find the domain of the rational function.

|  |  |
| --- | --- |
| a. $f\left(x\right)=\frac{3x-8}{x+4}$ | b. $g\left(x\right)=\frac{5x}{x^{2}-10x-24}$ |

# Objective 2: Simplifying Rational Expressions

Recall that a fraction is in simplest form if the numerator and denominator have no common factors other than $1$ or $-1$. The same is true for a rational expression. To **simplify** a rational expression means to write the expression so that the numerator and denominator have no common factors other than $1$ or $-1$.

Write each expression in simplest form.

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| --- | --- |
| a. $\frac{6}{15}$  | b. $\frac{x^{2}+5x+6}{x^{2}+8x+15}$ where $x\ne -3,-5$ |

To simplify a rational expression, first completely factor the numerator and denominator. Then, divide out factors common to the numerator and denominator.

**Fundamental Principle of Rational Expressions:**

For any rational expression $\frac{P}{Q}$ and any polynomial $R$, where $R\ne 0$,

$$\frac{PR}{QR}=\frac{P}{Q}⋅\frac{R}{R}=\frac{P}{Q}$$

since $\frac{R}{R}$ is equal to $1$.

For now, we assume that variables in a rational expression do not represent values that make the denominator $0$.

Simplify the rational expression.

|  |  |
| --- | --- |
| c. $\frac{6x-3x^{2}}{12x}$ | d. $\frac{x^{2}-4}{2-x}$ |

|  |  |
| --- | --- |
| e. $\frac{x^{3}-216}{3x-18}$ | f. $\frac{x^{2}+x}{x^{2}-8x-9}$ |