Section 7.4 Dividing Polynomials

# Objective 1: Dividing a Polynomial by a Monomial

Recall that a rational expression is the quotient of two polynomials. An equivalent form of a rational expression can be obtained by performing the indicated division. For example, the rational expression $\frac{10x^{3}-5x^{2}+20x}{5x}$ can be thought of as the polynomial $10x^{3}-5x^{2}+20x$ divided by the monomial $5x.$

To divide a polynomial by a monomial, divide each term in the polynomial by the monomial.

**Dividing a Polynomial by a Monomial:**

For monomials $a$, $b$, and $c$,

$\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$, where $c\ne 0$.

Divide.

|  |  |
| --- | --- |
| a. $\frac{10x^{3}-5x^{2}+20x}{5x}$ | b. $\frac{8m^{3}n^{2}+14m^{2}n}{2m^{2}n}$ |

# Objective 2: Dividing by a Polynomial

To divide a polynomial by a polynomial other than a monomial, we use **long division**. Polynomial long division is similar to long division of integers.

To illustrate the similarities between integer and polynomial long division, we will look at two analogous problems. Note that the expressions in part b evaluate to equal those in part a when $x=10$.

Divide.

|  |  |
| --- | --- |
| a. $176÷11$  | b. $\left(x^{2}+7x+6\right)÷(x+1)$ |

Divide.

|  |  |
| --- | --- |
| c. $(6x^{3}+10x^{2}-9x-15)÷\left(3x+5\right)$ | d. $(2x^{2}-3x-14)÷\left(2x-1\right)$ |

When an $x$ term is missing, use a $0$ coefficient placeholder for the missing term.

Divide.

|  |  |
| --- | --- |
| e. $(3x^{3}+2x^{2}+6)÷(x-2)$ | f. $(x^{4}+x^{3}-8x^{2}-4x+6)÷(x^{2}+1)$ |

# Objective 3: Using Synthetic Division to Divide a Polynomial by a Binomial

When a polynomial is divided by a binomial of the form $x-c$, a shortcut process called **synthetic division** may be used. With synthetic division, we find the quotient by performing algebraic operations on the coefficients only.

Divide using synthetic division.

|  |  |
| --- | --- |
| a. $(3x^{3}+2x^{2}+6)÷(x-2)$ | b. $(x^{3}+9x^{2}+2x-9)÷(x+4)$ |

# Objective 4: Using the Remainder Theorem to Evaluate Polynomials

Consider the polynomial function $P\left(x\right)=4x^{2}-15x+10$.

a. Find $P(3)$.

b. Use synthetic division to find the remainder when $P(x)$ is divided by $x-3$.

Notice in the preceding example that $P\left(3\right)=1$ and that the remainder when $P(x)$ is divided by $x-3$ is also $1$. This is no accident. This illustrates the **remainder theorem**.

**Remainder Theorem:**

For a polynomial function $P$, if $P(x)$ is divided by $x-c$, then the remainder is $P(c)$.

c. Consider the polynomial function $P\left(x\right)=3x^{4}+2x^{2}-9$. Use the remainder theorem to find $P(-3)$.

# Objective 5: Using the Remainder Theorem to Evaluate Polynomials

A consequence of the remainder theorem is that if a polynomial function $f$ is divided by $x-c$ and the remainder is zero, then $f\left(c\right)=0$. This means that $c$ is a solution of the equation $f\left(x\right)=0$, that $c$ is a zero of $f$, and that $x-c$ is a factor of $f$.

**Factor Theorem:** A polynomial function $f$ has a factor of $x-c$ if and only if $f\left(c\right)=0$.

Consider the function $f\left(x\right)=x^{3}-4x^{2}+8x-8$.

a. Determine the remainder when $f(x)$ is divided by $x-2$.

b. Is $x-2$ a factor of $f$?

c. Name one zero of the function $f$.

Consider the equation $2x^{3}-x^{2}-5x-2=0$.

d. Use synthetic division to show that $-1$ is a solution to the equation.

e. Find the solution set of the equation.