Section 8.2 Rational Exponents

# Objective 1: Understanding the Meaning of

Up to this point, we have applied properties of exponents strictly to expressions containing integer exponents. In this section, we will extend the properties of exponents to include **rational exponents** as well. We will begin with unit fraction exponents.

Suppose that . Then

.

Since , or . Because is a positive number, we can conclude that .

Now suppose that . Then

.

Since , . We can conclude that .

Notice that in each example, the denominator of the rational exponent corresponds to the index of the radical.

**Definition of :** If is an integer greater than and is a real number, then .

Rewrite each expression using a radical. Then evaluate.

|  |  |
| --- | --- |
| a.  | b.  |

Rewrite each expression using a radical. Then simplify if possible.

|  |  |
| --- | --- |
| c.  | d.  |

# Objective 2: Understanding the Meaning of

We will now look at rational exponents of the form . Consider the expression . Using properties of exponents, we can rewrite this expression in two ways and apply the definition of from above.

Notice that the denominator of the rational exponent corresponds to the index of the radical, and the numerator of the rational exponent indicates that the base is to be raised to the power of .

**Definition of :**

If and are integers greater than with in lowest terms, then

as long as is a real number.

Rewrite each expression using a radical. Then evaluate.

|  |  |
| --- | --- |
| a.  | b.  |

Rewrite each expression using a radical.

|  |  |
| --- | --- |
| c.  | d.  |

# Objective 3: Understanding the Meaning of

To complete the set of definitions for rational exponents, we define .

**Definition of :**

as long as is a nonzero real number.

Rewrite each expression using a radical. Then evaluate.

|  |  |
| --- | --- |
| a.  | b.  |

# Objective 4: Using Rules for Exponents to Simplify Expressions

Since the properties of exponents hold for rational exponents, we can use these properties to simplify expressions that contain rational exponents.

**Summary of Exponent Rules:**

If and are rational numbers, and and are numbers for which the expressions below exist, then the following properties are true.

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |

Multiply. Assume that all variables represent positive real numbers.

|  |  |
| --- | --- |
| a.  | b.  |

Use rational exponents to write as a single radical expression. Assume that all variables represent positive real numbers.

|  |  |
| --- | --- |
| c.  | d.  |

Use properties of exponents to simplify the expression. Write with positive exponents. Assume that all variables represent positive real numbers.

|  |  |
| --- | --- |
| e.  | f.  |