Section 8.5 Rationalizing Denominators and Numerators of Radical Expressions

# Objective 1: Rationalizing Denominators of Radical Expressions

When working with radical expressions such as $\frac{\sqrt{3}}{\sqrt{2}}$ , it is sometimes useful to write the expression either without a radical in the denominator or without a radical in the numerator.

The process for writing a radical expression as an equivalent expression but without a radical in the denominator is called **rationalizing the denominator** because our goal is to rewrite the expression so that the denominator is a rational number. For example, for the expression $\frac{\sqrt{3}}{\sqrt{2}}$ we can rationalize the denominator by multiplying both the numerator and the denominator by $\sqrt{2}$.

$$\frac{\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{3}}{\sqrt{2}}⋅\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}}{\sqrt{4}}=\frac{\sqrt{6}}{2}$$

Rationalize the denominator. Assume that all variables represent positive real numbers. Give answers in simplest form.

|  |  |
| --- | --- |
| a. $\frac{2}{\sqrt{3}}$ | b. $\frac{9}{\sqrt[3]{x^{2}}}$ |

|  |  |
| --- | --- |
| c. $\frac{12}{\sqrt{7x}}$ | d. $\frac{12}{\sqrt[3]{27x}}$ |

|  |  |
| --- | --- |
| e. $\frac{\sqrt{5}}{\sqrt{8}}$ | f. $\frac{\sqrt[3]{5}}{\sqrt[3]{4}}$ |

# Objective 2: Rationalizing Denominators Having Two Terms

Recall the difference of squares identity from Chapter 6.

$$\left(a+b\right)\left(a-b\right)=a^{2}-b^{2}$$

 The expressions $(a+b)$ and $(a-b)$ are called **conjugates** of each other. To rationalize a numerator or denominator that contain a radical expression that is the sum or difference of two terms, we use conjugates and the difference of squares identity.

For example, the conjugate of $(\sqrt{3}+\sqrt{2})$ is $(\sqrt{3}-\sqrt{2})$. We can rationalize the denominator of the expression $\frac{1}{\sqrt{3}+\sqrt{2} }$ by multiplying both the numerator and the denominator by $(\sqrt{3}-\sqrt{2})$.

$$\frac{1}{\sqrt{3}+\sqrt{2} }=\frac{1}{(\sqrt{3}+\sqrt{2}) }⋅\frac{\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)}=\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^{2}-\left(\sqrt{2}\right)^{2}}=\frac{\sqrt{3}-\sqrt{2}}{3-2}=\sqrt{3}-\sqrt{2}$$

Rationalize the denominator. Assume that all variables represent positive real numbers. Give answers in simplest form.

|  |  |
| --- | --- |
| a. $\frac{9}{2-\sqrt{7}}$ | b. $\frac{5}{\sqrt{x}+\sqrt{y}}$ |

|  |  |
| --- | --- |
| c. $\frac{\sqrt{11}-\sqrt{5}}{\sqrt{11}+\sqrt{5}}$ |  |

# Objective 3: Rationalizing Numerators

The process for writing a radical expression as an equivalent expression but without a radical in the numerator is called **rationalizing the numerator** because our goal is to rewrite the expression so that the numerator is a rational number. For example, for the expression $\frac{\sqrt{3}}{\sqrt{2}}$ we can rationalize the numerator by multiplying both the numerator and the denominator by $\sqrt{3}$.

$$\frac{\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{3}}{\sqrt{2}}⋅\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{9}}{\sqrt{6}}=\frac{3}{\sqrt{6}}$$

Rationalize the numerator. Assume that all variables represent positive real numbers. Give answers in simplest form.

|  |  |
| --- | --- |
| a. $\frac{\sqrt{9x}}{2}$ | b. $\frac{\sqrt[3]{9x}}{2}$ |

|  |  |
| --- | --- |
| c. $\frac{\sqrt{x}-4}{\sqrt{x}}$ |  |