Section 9.4 Zeros of Polynomial Functions

# Objective 1: Finding Zeros of Polynomial Functions

Recall that a **zero of a function** $f$ is any number $c$ such that $f\left(c\right)=0$. Zeros can be real or nonreal numbers. Real zeros correspond to the $x$-intercepts of the graph of $f$.

For example, the function $f\left(x\right)=(x-2)(x+2)(x-3i)(x+3i)$ has four zeros which can be found by setting the expression that defines $f$ equal to zero and applying the zero factor property.

$$\left(x-2\right)\left(x+2\right)\left(x-3i\right)\left(x+3i\right)=0$$

$x=2$ or $x=-2$ or $x=3i$ or $x=-3i$

The zeros of $f$ are $2, -2, 3i, $and $-3i$. The graph of $f$ has two $x$-intercepts at $2$ and $-2$.

We can sometimes find zeros of polynomial functions of degree greater than $2$ by using methods of factoring learned in previous sections.

Find all zeros of the polynomial function. Give the answers in exact form using simplified radicals and $i$ as needed.

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| a. $f\left(x\right)=x^{4}-x^{2}-20$ | b. $f\left(x\right)=x^{3}+x^{2}+x+1$ |

In section 7.4, we used the remainder and factor theorems to identify whether or not a given value was a zero of a given polynomial function. We will now revisit these theorems and use them to find zeros of polynomial functions of degree greater than $2$ when known factoring methods do not work.

c. One zero of the function $f\left(x\right)=9x^{3}+6x^{2}-95x+100$ is $-4$. Find the remaining zero(s) of $f$.

Consider the polynomial function $f\left(x\right)=x^{3}-4x^{2}+14x-20$.

d. Given that $2$ is one zero of $f$, find the remaining zero(s).

e. Rewrite the expression that defines $f$ as a product of linear factors.

# Objective 2: Writing Equations of Polynomial Functions with Given Zeros

Given the zeros of a polynomial function, it is possible to write an equation that satisfies those zeros. In order to write a unique equation, we need to know the value of the leading coefficient or one additional point on the graph.

For polynomial functions, complex nonreal zeros always occur in pairs. If one zero of a polynomial function is a complex, nonreal zero, then its conjugate is also a zero of the function. For example, if one zero of polynomial function $f$ is $5-2i$, then another zero of $f$ must be $5+2i$.

Write the equation of the degree three polynomial function that has a leading coefficient of $1$ if two of the zeros are $-5$ and $1+i$.