August 6, 2008

Math Tune-Up Workshop, Calculus Set 2

The workshop participants should attempt to solve the problems by themselves, without the help of the assistants. Only when they are unable to solve the problem, should they receive help. The word problems are the most important since they require not only skills in doing calculations, but also good understanding of the material. If there is not enough time to solve all problems, the word problems should be discussed before the time runs out.

1. Calculate the double integrals

   (a)
   $$\iint_R \frac{xy^2}{x^2+1}\,dA, \qquad R = \{(x,y)\,|\,0 \le x \le 1, -3 \le y \le 3\}$$

   (b)
   $$\iint_R x\sin(x+y)\,dA, \qquad R = [0,\pi/6]\times[0,\pi/3]$$

2. Find the volume of the solid bounded by the surface $z = 6 - xy$ and the planes $x = 2$, $x = -2$, $y = 0$, $y = 3$, and $z = 0$. If available, use Mathematica to draw the solid.

3. Find the volume of the solid bounded by the cylinders $z = x^2$, $y = x^2$, and the planes $z = 0$, $y = 4$.

4. Find the volume of the solid bounded by the cylinders $x^2 + y^2 = r^2$ and $y^2 + z^2 = r^2$. Draw the solid.

5. Evaluate the integral by reversing the order of integration

   (a) $\int_0^1 \int_{3y}^3 e^{x^2}\,dx\,dy$

   (b) $\int_0^3 \int_{y^2}^9 y\cos(x^2)\,dx\,dy$

6. Use a double integral to find the area of one loop of the rise $r = \cos 3\theta$.

7. Use polar coordinates to find the volume of the solid above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$. Solve this problem again using spherical coordinates.

8. The homogeneous lamina of density δ has the shape of a quarter disk of radius 2. Find the moment of inertia as the lamina is rotated about

   (a) the axis perpendicular to the lamina and passing through the center of the disk,

   (b) the axis perpendicular to the lamina and passing through the center of mass, and

   (c) the axis in the lamina that forms its axis of symmetry.

   If needed, use Mathematica to help you with integration or numerical integration. Before doing the calculations, can you guess which of the three moments of inertia would be the largest? Which would be the smallest?

9. Find the area of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

10. Find the surface of the solid created when the cylinder $y^2 + z^2 = 1$ intersects the cylinder $x^2 + z^2 = 1$.

11. Find the center of mass of the solid homogeneous hemisphere of radius a. Find the moment of inertia as the solid is rotated about its axis of symmetry.