Instructions. Answer each of the questions on your own paper, and be sure to show your work so that partial credit can be adequately assessed. Put your name on each page of your paper.

1. [**12** Points] Divide the following propositional formulas into groups satisfying the properties: (1) all propositional formulas in the same group are logically equivalent; (2) formulas in different groups are not equivalent.

   (a) $p \implies q$

   (b) $(\sim q) \implies (\sim p)$

   (c) $\sim (p \vee q)$

   (d) $p \vee (\sim q)$

   (e) $\sim (\sim p \wedge q)$

   (f) $(\sim p) \vee q$

2. [**10** Points] Recall that the Boolean set $\mathbb{B} = \{0, 1\}$. Define a Boolean function $f : \mathbb{B}^3 \to \mathbb{B}$ on the three Boolean variables p, q, and r by means of the formula

$$f(p, q, r) = 1 \text{ if and only if an odd number of } p,\ q,\ r \text{ have value } 1.$$

   (a) Fill in the truth table for f:

p	q	r	f
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

   (b) Express f using only $\sim$, $\vee$, and $\wedge$.

3. [**8** Points] Let $P(x, y)$ be the predicate: "Team x in Conference y has a winning record." Express each of the following using quantifiers and the predicate $P(x, y)$.

   (a) Every conference has at least one team with a winning record.

   (b) There is a conference in which no team has a winning record.

4. [**8** Points] Let $P(x)$ be the predicate "x is odd", and let $Q(x)$ be the predicate "x is twice an integer". Determine whether the following quantified statements are true (and, of course, explain your answer):

   (a) $(\forall x \in \mathbb{Z})(P(x) \implies Q(x))$.

   (b) $(\forall x \in \mathbb{Z})(P(x)) \implies (\forall x \in \mathbb{Z})(Q(x))$.

5. [**12** Points] Rewrite each of the following statements so that negations are applied exclusively to predicates (that is, so that no negation precedes a quantifier or an expression involving logical connectives).

   (a) $\sim \forall x \forall y P(x,\, y)$

   (b) $\sim \forall y \exists x P(x,\, y)$

   (c) $\sim \forall y \forall x (P(x,,y) \vee Q(x,\, y))$

   (d) $\sim (\exists x (\sim P(x)) \wedge \forall y Q(y))$

6. [**12** Points] If x is an integer, consider the statement: "If x is even, then x^2+1 is odd."

   (a) Write the converse statement.

   (b) Write the contrapositive statement.

   (c) Which, if any, of the original statement, the converse, or the contrapositive are true? (No proof required.)

7. [**8** Points] Let $f : X \to \mathbb{R}$ be a real valued function with domain X. In calculus you learned the definition of "f is a bounded function." Namely,

   The function f is bounded if there exists a positive number $M \in \mathbb{R}$ such that $|f(x)| \leq M$ for all $x \in X$.

   Write the negation of this definition, that is the definition of "f is an unbounded function," without using words like "not" or "it is not true that".

8. [**20** Points] For $n \geq 1$, let $F(n)$ be defined by the summation

   $$(*) \qquad F(n) = \sum_{j=1}^{n} \frac{1}{(2j-1)(2j+1)}.$$

   (a) Compute $F(1)$, $F(2)$, and $F(3)$ from the definition $(*)$.

   (b) Find the number A_n so that $F(n+1) = F(n) + A_n$. (In other words, what is the number that must be added to the sum $F(n)$ in order to get the sum $F(n+1)$.)

   (c) Prove by induction that

   $$F(n) = \frac{n}{2n+1}, \text{ for } n \geq 1.$$

   As usual, your argument should be written in complete sentences, and you must make clear where you are using your induction hypothesis. You may (and probably should) find your answers to earlier parts of this exercise to be of use in your argument.

9. [**10** Points] The following summation formulas were proved in class:

   $$\text{(a)} \sum_{j=0}^{n} q^n = \frac{q^{n+1}-1}{q-1},\ q \neq 1 \qquad \text{(b)} \sum_{j=1}^{n} j = \frac{n(n+1)}{2}.$$

   Assuming these results, (or any method you prefer), evaluate the following sums.

   (a) $\sum_{i=2}^{n} 7 \cdot 3^i$

   (b) $\sum_{k=1}^{n} (3k-1)$