Instructions. Answer each of the questions on your own paper, and be sure to show your work so that partial credit can be adequately assessed. Put your name on each page of your paper.

1. **[21 Points]** Let $A = \{a, b, c, d, e\}$, $B = \{1, a, b, 2, c\}$, and $C = \{3, c, a, 1, f\}$. Answer the following questions concerning these sets. Remember that the cardinality of a set means the number of elements in the set.

   (a) Write the set $A \cap B$.

   (b) Write the set $A \cup B$.

   (c) Write the set $A \setminus (B \cap C)$.

   (d) Write the set $(A \cap C) \times (C \setminus B)$.

   (e) What is the cardinality of $\mathcal{P}(A)$? Remember that $\mathcal{P}(A)$ denotes the power set of A.

   (f) What is the cardinality of $A \times C$?

   (g) Which set has more elements: $\mathcal{P}(A \times C)$ or $\mathcal{P}(A) \times \mathcal{P}(C)$?

2. **[6 Points]** If A, B, and C are arbitrary sets, draw a Venn diagram and shade the area corresponding the following set:

$$[(A \cap B) \setminus C] \cup [(A \cap C) \setminus B] .$$

3. **[12 Points]** How many subsets of the set $V = \{a, e, i, o, u, y\}$ contain exactly 4 elements? How many words of length 3 can be formed from the letters of V if no repetitions are allowed?

4. **[9 Points]** Which of the following are partitions of $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$? Explain your answers.

   (a) $\{\{1, 3, 5\}, \{1, 2, 6\}, \{4.7, 8\}\}$

   (b) $\{\{1, 3, 5\}, \{2, 6, 7\}, \{4, 8\}\}$

   (c) $\{\{1, 5\}, \{2, 6\}, \{4, 8\}\}$

5. **[10 Points]** How many functions are there from a set X with 5 elements to a set Y with 7 elements? How many of these functions are injective?

6. **[12 Points]** Let $f : A \to B$ be a function. Recall that the function f is said to be *surjective* (or *onto*) if for every $b \in B$ there is an $a \in A$ such that $f(a) = b$.

   (a) Give the definition of *injective* (or *one-to-one*) function by completing the sentence:

   The function f is injective if $\boxed{}$.

   Now suppose that $A = B = \mathbb{Z}$ = the set of integers, and f is given by the formula $f(x) = 4x + 7$. *The following two questions concern this specific function $f : \mathbb{Z} \to \mathbb{Z}$.*

   (b) Is f injective? Explain your answer using the definition above.

   (c) Is f surjective? Explain your answer using the definition above.

7. **[9 Points]** If the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x - 2$, find the inverse function $f^{-1} : \mathbb{R} \to \mathbb{R}$.

8. **[21 Points]** The following two permutations in $S(7)$ are given in two-rowed notation:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 & 7 & 2 & 1 \end{pmatrix} \text{ and } \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 2 & 1 & 3 \end{pmatrix}.$$

   (a) Compute the products $\sigma\tau$ and $\tau\sigma$. Express your answers in two-rowed notation. Are these two products equal?

   (b) Compute σ^{-1}. Again express your answer in two-rowed notation.

   (c) Write each of σ, τ, $\sigma\tau$ and $\tau\sigma$ as a product of disjoint cycles.