The first midterm exam will be on Thursday, February 15, 2007. The syllabus consists of Section 1 of Unit BF, Unit Lo, and Section 1 of Unit IS from the text. The table of contents lists the basic terms covered in each section and you should be sure that you know the meaning of all of those terms. Moreover, you should be able to do all of the assigned problems, both suggested and those that were turned in.

The following are a few problems that are representative of the type and difficulty of problems that might appear on your exam. The problems are listed randomly, and are not necessarily in any order reminiscent of the order that topics were covered in class.

1. (a) Consider the sequence $1, -2, 4, -8, 16, -32, \ldots$. Letting $a_0 = 1$, $a_1 = -2$, etc. give a general formula for a_i.
   (b) Give a formula for the sum $\sum_{i=0}^{n} a_i$.
2. Convert the propositional formula $p \implies (q \implies r)$ into an equivalent formula using only the connectives $\wedge$, $\vee$, and $\sim$.
3. Which of the following are logically equivalent to $p \implies q$? Give the label of all choices that apply. You need not show any work.
   (a) $\sim (q \implies p)$
   (b) $(\sim p) \implies (\sim q)$
   (c) $(\sim q) \implies (\sim p)$
   (d) $p \wedge \sim q$
   (e) $p \vee \sim q$
   (f) $(\sim p) \wedge q$
   (g) $(\sim p) \vee q$.
4. For each formula in the left column, find a logically equivalent formula in the right column. It is possible that some items in the right column will be used more than once, and it is certain that some items in the right column will not be used at all.

____	$(p \implies q) \vee (q \implies p)$	**a.**	p
____	$(p \implies q) \wedge (q \implies p)$	**b.**	$\sim p$
____	$p \implies (q \implies p)$	**c.**	$p \implies p$
____	$\sim (q \implies \sim p)$	**d.**	$p \wedge \sim p$
____	$\sim (\sim p \implies q)$	**e.**	$p \iff q$
		f.	$p \wedge q$
		g.	None of the above.

5. Let $P(x, y, z)$ be the predicate "$x = z - y$," where $x, y, z \in \mathbb{N}$. Determine whether each of the following statements are true:
   (a) $P(2, 5, 3)$
   (b) $(P(2, 5, 3) \wedge P(0, 4, 4)) \implies P(3, 6, 3)$
   (c) $(\exists x \in \mathbb{N})(\forall y \in \mathbb{N})P(x, x, y)$
   (d) $(\exists x \in \mathbb{N})(\forall y \in \mathbb{N})P(x, y, y)$
   (e) $(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})P(x, x, y)$

6. Determine which of the following are true statements. Recall that $\mathbb{R}$ denotes the real numbers, $\mathbb{R}^+$ denotes the positive real numbers, $\mathbb{Z}$ denotes the integers, and $\mathbb{Z}^+$ denotes the positive integers.
   (a) $\forall x \in \mathbb{R}^+, \exists y \in \mathbb{Z}^+$ such that $x = y + 1$.
   (b) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}$ such that $x = y + 1$.
   (c) $\exists x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}$, $x = y + 1$.
   (d) $\forall v \in \mathbb{R}^+$, $\exists u \in \mathbb{R}^+$ such that $uv < v$.
   (e) $\exists u \in \mathbb{R}^+$, such that $\forall v \in \mathbb{R}^+$, $uv < v$.
   (f) $\forall x \in \mathbb{Z}$ and $\forall y \in \mathbb{Z}$, $\exists z \in Z$ such that $z = x - y$.
   (g) $\forall x \in D$, $(P(x) \vee Q(x))$
   always has the same truth value as
   $(\forall x \in D, P(x)) \vee (\forall x \in D, Q(x))$.
   (h) $\forall x \in D$, $(P(x) \wedge Q(x))$
   always has the same truth value as
   $(\forall x \in D, P(x)) \wedge (\forall x \in D, Q(x))$.

7. Express each of the following assertions (semi-)formally. You may use quantifiers such as "$\forall$ rational x", "$\exists$ a positive integer y", Boolean connections, and predicates such as "$x < y$", "$x \leq y$", "$x = y$", "$x \neq y$", and functions like addition, subtraction, multiplication, and division.
   (a) There is a smallest positive integer.
   (b) There is no smallest positive rational number.
   (c) For any irrationals $x < y$, there is a rational z between x and y.

8. Define a sequence a_n as follows. Let $a_1 = 1$ and if $n \geq 1$ is a natural number, then $a_{n+1} = \frac{n^2}{n+1} a_n$. Prove that for all positive integers n,
$$a_n = \frac{(n-1)!}{n}.$$
   Recall that $m! = m(m-1)\cdots 2 \cdot 1$ is the product of all the natural numbers between 1 and m.

9. Consider the following (meaningless) statement: "If some ducks like reggae, then no cows wear neckties."

   (a) Write the converse:

   (b) Write the contrapositive:

10. Write each of the following sentences using propositional logic built up from these basic propositions:

    $M =$ "Mary sings"

    $B =$ "Beano cries"

    $Z =$ "Zena sleeps"

    (a) If Mary doesn't sing, then Beano cries and Zena sleeps.

    (b) Mary sings only if Beano cries.

    (c) For Zena to sleep, it is necessary that Mary sing.

    (d) Either Beano cries or Zena sleeps, but not both.

11. Use mathematical induction to show that

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1.$$

12. For each of the sums below, write it in summation notation. Then find and prove a formula for the sum in terms of n.

    (a) $3 + 7 + 11 + \cdots + (4n - 1)$

    (b) $1 + 5 + 9 + \cdots + (4n + 1)$

    (c) $-1 + 2 - 3 + 4 - \cdots - (2n - 1) + 2n$

    Hint: You may choose to prove that your formulas are valid either by use of induction, or it could be easier to use an already proved formula such as $\sum_{i=1}^{n} i = n(n+1)/2$.

13. Prove that $5^n + 5 < 5^{n+1}$ for all natural number $n \geq 1$.