1. Find the truth tables of each of the following compound statements.

   (a) $(\sim (p \wedge q)) \wedge (p \vee \sim q)$,

   (b) $[p \wedge (\sim p \vee q)] \vee [(\sim (p \wedge q)) \wedge (p \ \vee q)]$,

   (c) $(\alpha \wedge \sim \beta) \vee (\beta \wedge \sim \gamma) \vee (\gamma \wedge \sim \alpha)$.

2. Which of the following Boolean functions are equal? Prove using truth tables.

   (a) $(p \wedge \sim q) \vee (\sim p \wedge q)$,

   (b) $(p \vee q) \wedge (\sim (p \wedge q))$,

   (c) $\sim ((p \wedge q) \vee ((\sim p) \wedge (\sim q)))$.

3. Using the systematic approach illustrated in class and in the proof of Theorem 1, Page BF-5, express the Boolean functions given by the following truth tables using only $\sim$, $\vee$, and $\wedge$.

(a)

p	q	f
0	0	1
0	1	0
1	0	1
1	1	0

(b)

p	q	r	g
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

(c)

p	q	r	h
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

4. Classify each of the following statements as true, false, or not a valid mathematical statement. Explain your answer.

   (a) An integer is a rational number.

   (b) Let x be a real number.

   (c) $4 = 2 + 2$ and $7 < \sqrt{50}$.

   (d) 5 is an even integer and $16^{-1/4} = 1/2$.

   (e) 5 is an even integer or $16^{-1/4} = 1/2$.

   (f) $4 \neq 2 + 2 \implies 7 < \sqrt{50}$.

   (g) $4 = 2 + 2 \implies 7 > \sqrt{50}$.

5. Write down the negation of each of the following statements in clear and concise English. Do not use the expression "It is not the case that" in your answers.

   (a) Either $a^2 > 0$ or a is not a real number.

   (b) Every integer is divisible by a prime.

   (c) For every real number x, there is an integer n such that $n > x$.

   (d) There exists a planar graph that cannot be colored with at most four colors.

   (e) For all integers a and b, there exist integers q and r such that $b = qa + r$.

6. Write down the converse and the contrapositive of each of the following implications.

   (a) If $\frac{a}{b}$ and $\frac{b}{c}$ are integers, then $\frac{a}{c}$ is an integer.

   (b) If $x^2 = x + 1$, then $x = (1 + \sqrt{5})/2$ or $x = (1 - \sqrt{5})/2$.

   (c) If n is an odd integer, then $n^2 + n - 2$ is an even integer.

   (d) If $p(x)$ is a polynomial of odd degree, then $p(x)$ has at least one real root.

7. Which of the following is a tautology, a contradiction, or neither? (See page Lo-2 for the definitions of tautology and contradiction.)

   (a) $(p \wedge q) \vee (\sim (p \wedge q))$

   (b) $(p \wedge q \wedge r) \vee (p \wedge q \wedge (\sim r)) \vee (\sim (p \wedge q))$

   (c) $(p \vee q \vee r) \wedge (p \vee (\sim r)) \wedge (\sim p) \wedge (\sim q)$.

8. Given:

   - Predicates $I(x) =$"x is wise" and $E(x, y) =$"x knows y", and a
   - domain $P = \{a, b, c\}$, and
   - the truth set of $I(x)$ is $\{a\}$, and
   - the truth set of $E(x, y)$ is $\{(a, a), (b, b), (c, c), (a, c), (a, b)\}$. [Remember that the truth set of a predicate is the set of elements in the domain that make the predicate a true statement.]

   Answer the following questions:

   (a) Is the statement $(\forall x \in P)(\exists y \in P) : I(y) \implies E(x, x)$ true?

   (b) Is the statement $(\exists y \in P) : (\forall x \in P) \sim E(x, y) \vee \sim I(x)$ true?

   (c) What is the opposite (in colloquial English) of "everyone who is wise knows themselves"?

   (d) Convert the following from predicate logic to colloquial English: $\exists x \in P \forall y \in P : E(x, y \implies I(y)$.

   (e) Convert the following from English to predicate logic: "anyone who knows everybody is wise."