Exercises to turn in:

1. In each case determine whether the statement is true or false. (A calculator will be useful for the larger numbers.)

   (a) $40 \equiv 13 \pmod 9$ (b) $-29 \equiv 1 \pmod 7$
   (c) $8 \equiv 48 \pmod{14}$ (d) $-8 \equiv 48 \pmod{14}$
   (e) $7754 \equiv 357482 \pmod{3643}$ (f) $4015 \equiv 33303 \pmod{1295}$

2. In each case find all integers k making the statement true.

   (a) $4 \equiv 2k \pmod 7$ (b) $12 \equiv 3k \pmod{10}$
   (c) $3k \equiv k \pmod 9$ (d) $5k \equiv k \pmod{15}$

3. Find all incongruent solutions to each of the following congruences.

   (a) $7x \equiv 3 \pmod{15}$ (b) $6x \equiv 5 \pmod{15}$
   (c) $3x \equiv 1 \pmod{12}$ (d) $3x \equiv 1 \pmod{11}$
   (e) $15x \equiv 5 \pmod{17}$ (f) $5x \equiv 5 \pmod{18}$
   (g) $x^2 \equiv 1 \pmod 8$ (h) $x^2 \equiv 3 \pmod 7$

4. Determine the number of incongruent solutions for each of the following congruences. You need not write down the actual solutions.

   (a) $72x \equiv 47 \pmod{200}$

   (b) $1537x \equiv 2862 \pmod{6731}$

5. If $a \in \mathbb{Z}$ and $n > 1$ then a *multiplicative inverse of a mod n* is a solution of the congruence $ax \equiv 1 \pmod n$.

   (a) Explain how Theorem 8.1 (Page 57) of the handout shows that a has a multiplicative inverse mod n if and only if the greatest common divisor $(a, n) = 1$. Note that this theorem also shows you explicitly how to *find* the multiplicative inverse of a mod n, when it exists.

   (b) Find the inverse of 13 mod 35.

   (c) Find the inverse of 9 mod 16.

6. Let $n = d_k d_{k-1} \cdots d_2 d_1 d_0$ be the decimal representation of n. Recall that this means that

   $$n = d_k 10^k + d_{k-1} 10^{k-1} + \cdots + d_2 10^2 + d_1 10 + d_0,$$

   and each d_j is an integer between 0 and 9.

   (a) Show that $3|n$ if and only if $3|(d_0 + d_1 + \cdots + d_k)$.

   (b) Show that $11|n$ if and only if $11|(d_0 - d_1 + d_2 - d_3 + \cdots \pm d_k)$.

   Hint: Use the congruences $10 \equiv 1 \pmod 3$ and $10 \equiv -1 \pmod{11}$ and the rules of congruence arithmetic on Page 53 of the handout.