Instructions

- Answer each of the questions on your own paper, and be sure to show your work so that partial credit can be adequately assessed. Put your name on each page of your paper.
- You may use a scientific calculator, but it should not be needed. If you use a calculator you must still write out the operations done on the calculator to show that you know how to solve the problem.
- There are 5 problems, with a total of 100 points possible. A 10 point bonus problem is on the second page.

1. [**20 Points**] Compute the line integral $\int_C (x+2y)\,dx + x^2\,dy$ where C is the line segment from (0, 1) to (2, 0).

2. [**20 Points**] Let $\mathbf{F} = (2xy + z^3)\mathbf{i} + (x^2 + 2yz)\mathbf{j} + (y^2 + 3xz^2 - 1)\mathbf{k}$.

   (a) Verify that the vector field $\mathbf{F}$ is conservative.

   (b) Using a systematic method, find a potential function $f(x, y, z)$ such that $\mathbf{F} = \nabla f$.

   (c) Compute $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is any curve from (1, 2, 0) to (0, 3, 4).

3. [**20 Points**] Let C be the boundary of the rectangle with vertices (2, 1), (4, 1) (2, 4), (4, 4) and assume C is oriented counterclockwise. Use Green's theorem to evaluate the line integral
$$\int_C (y^2 + \cos x)\,dx + (x^2 - \sin y)\,dy.$$

4. [**20 Points**] Let $\mathbf{F} = (yz - xy)\mathbf{i} + (xz - xy)\mathbf{j} + (xz + yz)\mathbf{k}$.

   (a) Calculate $\operatorname{curl}\mathbf{F}$.

   (b) Calculate $\operatorname{div}\mathbf{F}$.

   (c) Show that $\iint_S (\operatorname{curl}\mathbf{F})\cdot\mathbf{n}\,dS = 0$ for any surface S contained in the plane $x+y+z = 1$.

   (d) Show that $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for any simple closed curve in the plane $x + y + z = 1$.

5. [**20 Points**] Let S be the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = -1$ and $z = 2$, and let $\mathbf{F} = x^3\mathbf{i} + xe^z\mathbf{j} + 3zy^2\mathbf{k}$. Use the divergence theorem to evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\,dS$, where $\mathbf{n}$ is the outward unit normal.

6. **Bonus Problem [10 Points]** The following are the main formulas of vector integral calculus, listed without hypotheses.

$$\int_a^b F'(x)\,dx = F(b) - F(a) \tag{1}$$

$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a)) \tag{2}$$

$$\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\,dA \tag{3}$$

$$\int_C \mathbf{F}\cdot\,d\mathbf{r} = \iint_S \operatorname{curl}\mathbf{F}\cdot\mathbf{n}\,dS \tag{4}$$

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_E \operatorname{div}\mathbf{F}\,dV \tag{5}$$

In the following table match the equation number with the appropriate theorem.

Equation Number	**Theorem**
	Green's Theorem
	Fundamental Theorem of Calculus
	Divergence Theorem
	Fundamental Theorem for Line Integrals
	Stokes' Theorem