1. Let $\mathbf{F}(x, y) = xy\mathbf{i} + (y-3x)\mathbf{j}$, and let C be the curve $\mathbf{r}(t) = t\mathbf{i} + (3t-t^2)\mathbf{j}$ for $0 \le t \le 2$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$.

2. Evaluate the line integral $\int_C \frac{x^3}{y}\, ds$, where C is the portion of the graph $y = x^2/2$ for $0 \le x \le 2$.

3. Let $\mathbf{F}(x, y) = (ax^2y + y^3 + 1)\mathbf{i} + (2x^3 + bxy^2 + 2)\mathbf{j}$ be a vector field, where a and b are constants.

   (a) Find the values and a and b for which $\mathbf{F}$ is conservative.

   (b) For these values of a and b, find $f(x, y)$ such that $\mathbf{F} = \nabla f$.

   (c) Still using the values of a and b from part (a), compute $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve C such that $x = e^t \cos t$, $y = e^t \sin t$, $0 \le t \le \pi$.

4. Let $\mathbf{F}(x, y, z) = (\cos x + 2y^2 + 5yz)\mathbf{i} + (4xy + 5xz)\mathbf{j} + (5xy + 3z^2)\mathbf{k}$ on $\mathbb{R}^3$. Show that $\mathbf{F}$ is conservative, and find a potential function for $\mathbf{F}$.

5. Verify that the vector field $\mathbf{F}(x, y) = (y^2 - y\cos x)\mathbf{i} + (2xy - \sin x + 1)\mathbf{j}$ is conservative. Then compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve $x = t^3 - t$, $y = 1 + t^2$, $1 \le t \le 2$.

6. Let C be the circle $x^2 + y^2 = 100$, oriented counterclockwise. Find

$$\int_C (2xy + e^x)\, dx + (x^2 - \sin y + 3x)\, dy.$$

7. Let C be the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \dfrac{x}{a^2}\mathbf{i} + \dfrac{y}{b^2}\mathbf{j}$.

8. Let C be the positively oriented closed curve formed by the parabola $y = x^2$ running from $(-1, 1)$ to $(1, 1)$ and by a straight line running back from $(1, 1)$ to $(-1, 1)$. Evaluate the line integral

$$\int_C y^2\, dx + 4xy\, dy$$

   in two ways:

   (a) directly, and

   (b) by using Green's theorem.

9. Let $\mathbf{F}(x, y, z) = y\mathbf{i} + z\mathbf{j} - 2x\mathbf{k}$ and let C be a simple closed curve in the plane $x + y + z = 1$. Show that $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$.

10. Let Σ be the unit sphere centered at the origin, with the outward normal orientation, and let $\mathbf{F}(x, y, z) = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$. Compute the integral:

$$\iint_\Sigma \mathbf{F} \cdot \mathbf{n}\, dS.$$

   Hint: Use the divergence theorem.

11. Let f be a scalar field (i.e., function) and $\mathbf{F}$ a vector field. Determine if each of the following expressions is meaningful. If it is meaningful, indicate whether it is a scalar field or a vector field.

Expression	Meaningful	Scalar	Vector
∇f			
$\operatorname{div} \mathbf{F}$			
$\operatorname{curl} f$			
$\operatorname{curl}(\nabla f)$			
$\nabla \mathbf{F}$			
$\operatorname{div}(\nabla \mathbf{F})$			