1. A function $u(x, y)$ is *harmonic* if it satisfies the partial differential equation $u_{xx}+u_{yy} = 0$. Determine if the function $u(x, y) = x^4 - 6x^2y^2 + y^4$ is harmonic.

2. Let $w = u^2v + 2v^3 - u + 1$, where $u = (x - y^2)^3$ and $v = x^2 - y + 1$. Compute the partial derivatives w_x and w_y.

3. Find the rate of change of $f(x, y) = 3x^4 - xy + y^3$ at the point $P(1, 2)$ in the direction of the vector $\mathbf{a} = 2\mathbf{i} - \mathbf{j}$.

4. Let S be the surface defined by the equation
$$z = x^2y + xy^2 - 3y^2.$$
   (a) Find the equation of the tangent plane to S at the point $P = (2, 1, 3)$.
   (b) Give a formula approximating the change Δz in z if x and y change by small amounts Δx and Δy.
   (c) Approximate the value of z at the point $(2.01, 1.01)$.

5. Let S be the surface
$$\frac{x^2}{1} + \frac{y^2}{4} + \frac{z^2}{9} = 1.$$
   (a) Find a normal vector to the surface S at $\left(\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \sqrt{3}\right)$ on S.
   (b) At which point(s) on S is the vector $\langle 1, 1, 1\rangle$ normal to S?

6. Find *all* critical points of $f(x, y) = x^3 + y^4 - 6x - 2y^2$. Apply the second derivative test to each point and determine whether it is a local maximum, local minimum, or saddle point, or that the test fails.

7. Use Lagrange multipliers to find the global maximum value and global minimum value of the function $f(x, y, z) = xyz$ subject to the constraint $g(x, y, z) = 1$ where $g(x, y, z) = \frac{x^2}{1} + \frac{y^2}{4} + \frac{z^2}{9}$.

8. Evaluate:
   (a) $\displaystyle\int_0^1 \int_1^{e^y} \frac{y}{x}\, dx\, dy$
   (b) $\displaystyle\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+1}\, dx\, dy$. (*Hint:* Sketch the domain and reverse the order of integration.)

9. Evaluate $\displaystyle\iiint_E z\, dV$ if E is the region defined by the inequalities $y \le z \le x$; $0 \le y \le x$; $0 \le x \le 1$.

10. Use spherical coordinates to evaluate $\displaystyle\iint_C x\, dV$ if E is the region defined by the inequalities $x^2 + y^2 + z^2 \le 1$; $x \ge 0$; $y \ge 0$; $z \ge 0$.

11. Let C_1 be the line segment from (0, 0) to (1, 0), C_2 the arc of the unit circle running from (1, 0) to (0, 1) and let C_3 be the line segment from (0, 1) to (0, 0). Let C be the simple closed curve formed by C_1, C_2, and C_3, and let $\mathbf{F} = x^3\mathbf{i} + y^2x\mathbf{j}$. Calculate

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

in two ways:

(a) Directly,

(b) using Green's theorem.

12. Let $\mathbf{F} = (6x + y + z)\mathbf{i} + (x - z + 7)\mathbf{j} + (x - y - 3z^2)\mathbf{k}$. Show that $\mathbf{F}$ is conservative and find a potential function for $\mathbf{F}$.

13. Let E be the 3-dimensional region described by the inequalities $x \geq 0$, $y \geq 0$, $x+y \leq 2$, and $0 \leq z \leq 3$. Let S be the entire boundary of E (all five faces)and let

$$\mathbf{F} = e^{-y^2}\mathbf{i} + \sin(e^x)\mathbf{j} + z^2\mathbf{k}.$$

Compute $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$, where the normal $\mathbf{n}$ is oriented outward.

Hint: Use a theorem instead of trying to compute it directly.