1. Evaluate the following limit or determine that it does not exist:

$$\lim_{(x,\,y)\to(0,\,2)} \left(\frac{1}{2xy} - \frac{1}{xy(x+2)} \right).$$

2. Show that the function $w = \ln(x^2 + y^2)$ satisfies the two-dimensional Laplace equation $w_{xx} + w_{yy} = 0$.

3. Let $f(x,\, y) = 3xy^2 - x - y$.

   (a) Find ∇f at $(2,\, 1)$.

   (b) Find the equation of the tangent plane to the graph of $f(x,\, y)$ at the point $(2,\, 1,\, 3)$.

   (c) Use linear approximation to estimate the value of $f(2.1,\, 0.9)$.

   (d) Find the directional derivative of f at $(2,\, 1)$ in the direction $\mathbf{i} + \mathbf{j}$.

4. Let $w = f(u,\, v)$ where $u = x^2 - y^2$ and $v = 2xy$.

   (a) Using the chain rule, express

   $$\frac{\partial w}{\partial x} \qquad \text{and} \qquad \frac{\partial w}{\partial y}$$

   in terms of x, y, f_u and f_v.

   (b) Assume that w solves the differential equation $uw_u + vw_v = 1$. Find $xw_x + yw_y$.

5. Suppose that the equation $x^2yz^3 + xz^2 + z + 2 = 0$ has a differentiable solution for z as a function of x and y. Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ at the point $(1,\, 2,\, -1)$.

6. Let $f(x,\, y) = -3x^2 - 4xy - y^2 - 12y + 16x$.

   (a) Find all the critical points of $f(x,\, y)$, and determine what type each critical point is.

   (b) Find the global maximum and minimum of $f(x,\, y)$ over the triangle with sides $x = 0$, $y = 0$ and $y + x = 4$.

7. Use Lagrange Multipliers to find the global *maximum and minimum* values of $f(x,\, y) = x^2 + 2y^2 - 4y$ subject to the constraint $x^2 + y^2 = 9$.