1. Evaluate the line integral $\int_C xy^4\,ds$ where C is the right half of the circle $x^2+y^2=25$ (i.e., the part of the circle with $x \ge 0$).

   ▶ **Solution.** Parametrize the half circle by $x = 5\cos t$, $y = 5\sin t$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$. Then

   $$ds = \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = \sqrt{(-5\sin t)^2 + (5\cos t^2)}\,dt = 5\,dt$$

   and

   $$\begin{aligned}\int_C xy^4\,ds &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (5\cos t)(5\sin t)^4\,5\,dt \\ &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 5^6\cos t\sin^4 t\,dt \qquad \text{(use substitution } u = \sin t) \\ &= 5^6\frac{\sin^5 t}{5}\bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 5^5\cdot 2 = 6250.\end{aligned}$$

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2. Let $\mathbf{F} = x^2y^3\mathbf{i} - y\sqrt{x}\mathbf{j}$ and let C be the curve given by $\mathbf{r}(t) = t^2\mathbf{i} - t^3\mathbf{j}$ for $0 \le t \le 1$. Compute the line integral

   $$\int_C \mathbf{F}\cdot d\mathbf{r}.$$

   ▶ **Solution.**

   $$\begin{aligned}\int_C \mathbf{F}\cdot d\mathbf{r} & \int_0^1 \left((t^2)^2(-t^3)^3\mathbf{i} - (-t^3)\sqrt{t^2}\mathbf{j}\right)\cdot(2t\mathbf{i} - 3t^2\mathbf{j})\,dt \\ &= \int_0^1 \left(-2t^{14} - 3t^6\right)\,dt \\ &= \left(-\frac{2t^{15}}{15} - \frac{3t^7}{7}\right)\bigg|_0^1 = -\frac{59}{105}.\end{aligned}$$

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