Instructions. Answer each of the questions on your own paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* Put your name on each page of your paper. A short table of Laplace Transforms and a table of common exact values of trigonometric functions is included on Page 2.

1. [16 Points Each] Solve each of the following initial value problems. Be sure to show all of your work.

   (a) $y' + \dfrac{t}{y} = 0, \quad y(1) = -3$

   (b) $y' - 2y = 2e^{5t} + 5e^{2t}, \quad y(0) = -3$

   (c) $y' - \dfrac{2}{t}y = 6t^4, \quad y(1) = -3$

2. [18 Points]

   (a) State Euler's formula for the complex exponential $e^{i\theta}$:

   $$\boxed{e^{i\theta} =}$$

   (b) Write the complex number $4e^{-i\pi/6}$ in rectangular form $x + iy$.

   (c) Determine the polar expression $z = re^{i\theta}$ (i.e., find r and θ) for the complex number $z = 1 + \sqrt{3}i$.

3. [18 Points] Compute the Laplace transform of each of the following functions.

   (a) $f(t) = t^3 - e^{-9t} + 5$

   (b) $g(t) = e^{-t}\cos 2t$

   (c) $h(t) = t^4 e^{-3t}$

4. [16 Points] A 400 gallon tank is initially full of brine which contains 60 pounds of salt. A solution containing 0.5 pounds of salt per gallon enters the tank at a rate of 6 gallons per minute. A drain is opened at the bottom of the tank through which the well stirred solution leaves the tank at the same rate of 6 gallons per minute. Let $y(t)$ denote the amount of salt (in pounds) which is in the tank at time t.

   (a) What is $y(0)$?

   (b) Write the differential equation that $y(t)$ must satisfy.

   (c) Solve the differential equation to find $y(t)$.

   (d) How much salt is in the tank after 1 hour?

Exam I Supplementary Sheet

A Short Table of Laplace Transforms

1.	$\mathcal{L}\{af(t)+bg(t)\}(s)$	$=$	$aF(s)+bG(s)$
2.	$\mathcal{L}\{e^{at}f(t)\}(s)$	$=$	$F(s-a)$
3.	$\mathcal{L}\{t^n\}(s)$	$=$	$\dfrac{n!}{s^{n+1}}$
4.	$\mathcal{L}\{e^{at}\}(s)$	$=$	$\dfrac{1}{s-a}$
5.	$\mathcal{L}\{t^n e^{\alpha t}\}(s)$	$=$	$\dfrac{n!}{(s-\alpha)^{n+1}}$
6.	$\mathcal{L}\{\cos bt\}(s)$	$=$	$\dfrac{s}{s^2+b^2}$
7.	$\mathcal{L}\{\sin bt\}(s)$	$=$	$\dfrac{b}{s^2+b^2}$
8.	$\mathcal{L}\{e^{at}\cos bt\}(s)$	$=$	$\dfrac{s-a}{(s-a)^2+b^2}$
9.	$\mathcal{L}\{e^{at}\sin bt\}(s)$	$=$	$\dfrac{b}{(s-a)^2+b^2}$

Common trigonometric values

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1