Instructions. Answer each of the questions on your own paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* Put your name on each page of your paper. A short table of integrals is included on Page 2.

1. **[18 Points]** Solve the initial value problem: $y' = 4t^3y^2, \quad y(0) = 1/4$

2. **[18 Points]** Find the general solution of: $y' - 3y = 5e^{3t} + e^t$

3. **[18 Points]** Solve the initial value problem: $ty' + 4y = t, \quad y(1) = -1$

4. **[18 Points]** Solve the initial value problem: $(4t + 4y + 3) + (4t - 6y - 2)y' = 0$, $y(2) = 1$

5. **[12 Points]** Apply Picard's method to compute the approximations $y_1(t)$, $y_2(t)$, and $y_3(t)$ to the solution of the initial value problem

$$y' = 2y - t, \quad y(0) = 0.$$

6. **[16 Points]** A 2000 gallon tank is initially full of brine which contains 100 pounds of salt. A solution containing 3.0 pounds of salt per gallon enters the tank at a flow rate of 4 gallons per minute. A drain is opened at the bottom of the tank through which the well stirred solution leaves the tank at the same flow rate of 4 gallons per minute. Let $y(t)$ denote the amount of salt (in pounds) which is in the tank at time t.

   (a) What is $y(0)$? That is, how much salt is in the tank at time $t = 0$?

   (b) Find the amount $y(t)$ of salt in the tank for all times t.

   (c) How much salt is in the tank after 30 minutes?

   (d) What is $\lim_{t\to\infty} y(t)$?

Exam I Supplementary Sheet

Some Integral Formulas

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$ (if $n \neq -1$)
2. $\int \frac{1}{x}\,dx = \ln|x| + C$
3. $\int \frac{1}{a+bx}\,dx = \frac{1}{b}\ln|a+bx| + C \quad (b \neq 0)$
4. $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \quad (a > 0)$
5. $\int \frac{1}{x(a+bx)}\,dx = \frac{1}{a}\ln\left|\frac{x}{a+bx}\right| + C$
6. $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$
7. $\int \ln x\,dx = x\ln x - x + C$
8. $\int xe^{ax}\,dx = \frac{xe^{ax}}{a} - \frac{e^{ax}}{a^2} + C$