Math 2065 Section 2
Review Exercises for Exam 2

Exam 2 will be on Friday, October 12, 2018. The syllabus for Exam 2 is Sections 2.1 – 2.7 of Chapter 2, Sections 3.1 – 3.4 of Chapter 3, and Sections 4.1 – 4.3 of Chapter 4. You should review all of the assigned exercises in these sections. Note that Section 2.2 contains the main formulas for computation of Laplace transforms. A Laplace transform table will be provided with the test. Following is a brief list of terms, skills, and formulas with which you should be familiar.

- Know how to use all of the Laplace transform formulas developed in Section 2.2 to be able to compute the Laplace transform of elementary functions.
- Know how to use partial fraction decompositions to be able to compute the inverse Laplace transform of any proper rational function. The key recursion algorithms for computing partial fraction decompositions are Theorem 1 (Page 129) for the case of a real root in the denominator, and Theorem 1 (Page 143) for a complex root in the denominator. Here are the two results:

  Theorem 1 (Linear Partial Fraction Recursion). *Let $P_0(s)$ and $Q(s)$ be polynomials. Assume that a is a number such that $Q(a) \neq 0$ and n is a positive integer. Then there is a unique number A_1 and polynomial $P_1(s)$ such that*

  $$\frac{P_0(s)}{(s-a)^n Q(s)} = \frac{A_1}{(s-a)^n} + \frac{P_1(s)}{(s-a)^{n-1}Q(s)}.$$

  The number A_1 and polynomial $P_1(s)$ are computed as follows:

  $$A_1 = \left.\frac{P_0(s)}{Q(s)}\right|_{s=a} \quad \text{and} \quad P_1(s) = \frac{P_0(s) - A_1 Q(s)}{s-a}.$$

  Theorem 2 (Quadratic Partial Fraction Recursion). *Let $P_0(s)$ and $Q(s)$ be polynomials. Assume that $a + bi$ is a complex number with nonzero imaginary part $b \neq 0$ such that $Q(a+bi) \neq 0$ and n is a positive integer. Then there is a unique linear term $B_1 s + C_1$ and polynomial $P_1(s)$ such that*

  $$\frac{P_0(s)}{((s-a)^2+b^2)^n Q(s)} = \frac{B_1 s + C_1}{((s-a)^2+b^2)^n} + \frac{P_1(s)}{((s-a)^2+b^2)^{n-1}Q(s)}.$$

  The linear term $B_1 s + C_1$ and polynomial $P_1(s)$ are computed as follows:

  $$B_1 s + C_1|_{s=a+bi} = \left.\frac{P_0(s)}{Q(s)}\right|_{s=a+bi} \quad \text{and} \quad P_1(s) = \frac{P_0(s) - (B_1 s + C_1)Q(s)}{(s-a)^2+b^2}.$$

- Know how to apply the input derivative principle (Corollary 8, Page 116) to compute the solution of an initial value problem for a constant coefficient linear differential equation with elementary forcing function. See Section 2.1.
- Know how to use the *characteristic polynomial* to be able to solve constant coefficient homogeneous linear differential equations. (See Algorithm 6, Page 232, and Algorithm 3, Page 286.)
- Know how to use the *method of undetermined coefficients* to find a particular solution of the constant coefficient linear differential equation

  $$q(\boldsymbol{D})y = f(t)$$

  where the forcing function $f(t)$ is an exponential polynomial, i.e., $f(t)$ is a sum of functions of the form $ct^k e^{at}\cos bt$ and $dt^k e^{at}\sin bt$ for various choices of the constants a, b, c, and d, and non-negative integer k. (See Algorithm 4, Page 239 and Algorithm 3, Page 295.)

The following is a small set of exercises of types identical to those already assigned.

1. Compute the Laplace transform of each of the following functions using the Laplace transform Tables. (A table of Laplace Transforms will be provided to you on the exam.)
   (a) $3t^3 - 2t^2 + 7$
   (b) $e^{-3t} + \sin\sqrt{2}t$
   (c) $-8 + \cos(t/2) + 5\sin 2t$
   (d) $7e^{2t}\cos 3t - 2e^{7t}\sin 5t$
   (e) $(2 - t^2)e^{-5t}$
   (f) $t\cos 6t$
   (g) $t^2\cos at$ where a is a constant

2. Find the inverse Laplace transform of each of the following functions. You may use the Laplace Transform Tables.
   (a) $\dfrac{7}{(2s+3)^3}$
   (b) $\dfrac{s+2}{s^2-3s-4}$
   (c) $\dfrac{s}{(s+4)^2+4}$
   (d) $\dfrac{1}{s^2-10s+9}$
   (e) $\dfrac{2s+18}{s^2+25}$

(f) $\dfrac{4s-3}{s^2-6s+25}$

(g) $\dfrac{1}{s(s^2+4)}$

(h) $\dfrac{1}{s^2(s+1)^2}$

3. Solve each of the following differential equations by means of the Laplace transform:

   (a) $y'' - 3y' + 2y = 4, \qquad y(0) = 2, \quad y'(0) = 3$

   (b) $y'' + 4y = 6\sin t, \qquad y(0) = 6, \quad y'(0) = 0$

4. Using the Laplace transform, find the solution of the following differential equations with initial conditions $y(0) = 0$, $y'(0) = 0$:

   (a) $y'' - y = 2\sin t$

   (b) $y'' + y = \sin 4t$

   (c) $y'' + y' = 1 + 2t$

   (d) $y'' + 4y' + 3t = 6$

   (e) $y'' + y = 2 + 2\cos t$

5. Solve each of the following homogeneous linear differential equations, using the techniques of Chapters 3 and 4 (Characteristic equation and calculation of the standard basis $\mathcal{B}_q$.)

   (a) $y'' + 3y' + 2y = 0$

   (b) $y'' + 6y' + 13y = 0$

   (c) $y'' + 6y' + 9y = 0$

   (d) $y'' - 2y' - y = 0$

   (e) $8y'' + 4y' + y = 0$

   (f) $2y'' - 7y' + 5y = 0$

   (g) $2y'' + 2y' + y = 0$

   (h) $y'' + .2y' + .01y = 0$

   (i) $y'' + 7y' + 12y = 0$

   (j) $y'' + 2y' + 2y = 0$

   (k) $y''' + 2y'' - 15y' = 0$

   (l) $y''' + 2y'' - 8y' = 0$

   (m) $y''' - 2y'' - 3y' = 0$

   (n) $y''' - 7y' + 6y = 0$

(o) $y''' - 3y'' - y' + 3y = 0$
(p) $4y''' - 10y' + 12y = 0$
(q) $y^{(4)} - 5y'' + 4y = 0$

6. Find the general solution of the constant coefficient homogeneous linear differential equation with the given characteristic polynomial $q(s)$.

   (a) $q(s) = (s-1)(s+3)(s-5)$
   (b) $q(s) = s^3 - 1$
   (c) $q(s) = (s^2-2)^2$
   (d) $q(s) = s^3 - 3s^2 + s + 5$
   (e) $q(s) = s^4 + 3s^2 - 4$
   (f) $q(s) = s^4 + 5s^2 + 4$
   (g) $q(s) = (s^2+1)^3$
   (h) $q(s)$ has degree 4 and has roots $\sqrt{2}$ with multiplicity 2 and $2 \pm 3i$ with multiplicity 1.
   (i) $q(s)$ has degree 5 and roots 0 with multiplicity 3 and $1 \pm \sqrt{3}$ with multiplicity 1.

7. Solve each of the following initial value problems. You may (and should) use the work already done in exercise 5.

   (a) $y'' + 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = -3.$
   (b) $y'' + 6y' + 13y = 0, \quad y(0) = 0, \quad y'(0) = -1.$
   (c) $y'' + 6y' + 9y = 0, \quad y(0) = -1, \quad y'(0) = 5.$
   (d) $y'' - 2y' - y = 0, \quad y(0) = 0, \quad y'(0) = \sqrt{2}.$
   (e) $y'' + 2y' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 2$

8. Use the method of undetermined coefficients (See Section 3.4) to find the general solution of each of the following differential equations.

   (a) $y'' - 3y' - 4y = 30e^t$
   (b) $y'' - 3y' - 4y = 30e^{4t}$
   (c) $y'' - 3y' - 4y = 20\cos t$
   (d) $y'' - 2y' + y = t^2 - 1$
   (e) $y'' - 2y' + y = 3e^{2t}$
   (f) $y'' - 2y' + y = 4\cos t$
   (g) $y'' - 2y' + y = 3e^t$
   (h) $y'' - 2y' + y = te^t$