Math 2065 Section 2
Final Exam Review Sheet

The final exam will be on Thursday, December 6, from 7:30 – 9:30 AM in the normal classroom. The exam is closed book, but you will be provided with the usual table of Laplace transforms. The final exam is comprehensive, and thus any of the material we covered is a valid source for questions. You should collect each of the review sheets and exams as sources for study for the final exam. Each of these is posted on the class web site, in case you have misplaced them. A good strategy for study is to do the review sheets and exams *without* looking at the answers. If you then compare with the answer sheets, you can identify the areas in which you need additional work. Some additional review exercises are included here, of exactly the same type as found in your text and on the previous review sheets.

Review Exercises

Solve each of the following differential equations.

1. $y' = t - 2y$
2. $y' = t - 4ty$
3. $y' + \frac{4}{t}y = t^4$
4. $yy' = (t-1)^2$
5. $y' = 1 + t + y^2 + ty^2$
6. $y'' - 3y' + 2y = 0$
7. $y'' + 2y' + 2y = 0$
8. $y'' + 4y' + 4y = 0$
9. $y'' + 6y' + 9y = 0$
10. $y'' - 6y' + 13y = 0$
11. $y'' + 16y = 0$
12. $y'' - 2y' - 3y = 0, \quad y(0) = 0, \quad y'(0) = 1$
13. $y'' + 6y' + 13y = 0, \quad y(0) = 1, \quad y'(0) = -1$
14. $y'' - 2y' - y = 0$
15. $y'' + 2y' - 15y = 0$

16. $t^2y'' + 2ty' - 6y = 0$

17. $3t^2y'' + 11ty' - 3y = 0$

18. $t^2y'' + 9ty' + 17y = 0$

19. $t^2y'' - 3ty' + 4y = 0$

20. $y'' - 2y' + y = 3e^{2t}$

21. $y'' + 2y' + y = 2e^{-t}$

22. $y'' - y' - 2y = -9e^{-t}$

23. $y'' - 2y' + y = \dfrac{e^t}{t^5}$

24. $y'' + \dfrac{1}{t}y' - \dfrac{1}{t^2}y = \ln t$, $(t > 0)$.

Find the Laplace transform of each of the following functions.

25. t^2e^{-9t}

26. $e^{2t} - t^3 + t^2 - \sin 5t$

27. $t\cos 6t$

28. $2\sin t + 3\cos 2t$

29. $e^{-5t}\sin 6t$

30. $t^2\cos at$ where a is a constant

31. $f(t) = \begin{cases} 1 & \text{if } 0 \le 0 < 2, \\ -1 & \text{if } 2 \le t < 4, \text{ and} \\ 0 & \text{if } t \ge 4. \end{cases}$

32. $f(t) = (t^2 - 100)h(t - 10)$

Find the inverse Laplace transform of each of the following functions.

33. $\dfrac{1}{s^2 - 10s + 9}$

34. $\dfrac{2s - 18}{s^2 + 9}$

35. $\dfrac{2s + 18}{s^2 + 25}$

36. $\dfrac{s+3}{s^2+5}$

37. $\dfrac{s-3}{s^2-6s+25}$

38. $\dfrac{1}{s(s^2+4)}$

39. $\dfrac{1}{s^2(s+1)^2}$

40. $\dfrac{1-e^{-s}}{s}$

41. $\dfrac{1+e^{-\pi s}}{s^2+1}$

42. Let $A = \begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix}$

    (a) Compute $(sI - A)$ and $(sI - A)^{-1}$.

    (b) Find $\mathcal{L}^{-1}((sI - A)^{-1})$.

    (c) What is e^{At}?

    (d) Solve the system $\mathbf{y}' = A\mathbf{y}$, $\mathbf{y}(0) = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

43. Solve the matrix differential equation $\mathbf{y}' = A\mathbf{y}$ where $A = \begin{bmatrix} 3 & -1 \\ -5 & -1 \end{bmatrix}$.

44. Solve the initial value problem:

$$\mathbf{y}' = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \mathbf{y}, \quad \mathbf{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

45. Solve the initial value problem:

$$\mathbf{y}' = \begin{bmatrix} 0 & -3 \\ 3 & 6 \end{bmatrix} \mathbf{y}, \quad \mathbf{y}(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

Answers

1. $y = \frac{1}{2}t - \frac{1}{4} + ce^{-2t}$
2. $4y = 1 + ce^{-2t^2}$
3. $y = \frac{c}{t^4} + \frac{1}{9}t^5$
4. $3y^2 - 2(t-1)^3 = c$
5. $\arctan y - t - \frac{t^2}{2} = c$
6. $y = c_1e^t + c_2e^{2t}$
7. $y = e^{-t}(c_1 \cos t + c_2 \sin t)$
8. $y = c_1e^{-2t} + c_2te^{-2t}$
9. $y = c_1e^{-3t} + c_2te^{-3t}$
10. $y = c_1e^{3t} \cos 2t + c_2e^{3t} \sin 2t$
11. $y = c_1 \cos 4t + c_2 \sin 4t$
12. $y = \frac{1}{4}(e^{3t} - e^{-t})$
13. $y = e^{-3t}(\cos 2t + \sin 2t)$
14. $y = c_1e^{(1+\sqrt{2})t} + c_2e^{(1-\sqrt{2})t}$
15. $y = c_1e^{3t} + c_2e^{-5t}$
16. $y = c_1t^{-3} + c_2t^2$
17. $y = c_1t^{1/3} + c_2t^{-3}$
18. $y = t^{-4}(c_1 \cos(\ln|t|) + c_2 \sin(\ln|t|))$
19. $y = c_2t^2 + c_2t^2 \ln|t|$
20. $y = c_1e^t + c_2te^t + 3e^{2t}$
21. $y = c_1e^{-t} + c_2te^{-t} + t^2e^{-t}$
22. $y = c_1e^{-t} + c_2e^{2t} + 3te^{-t}$
23. $y = c_1e^t + c_2te^t + \frac{1}{12}t^{-3}e^t$
24. $y = c_1t + c_2t^{-1} + \frac{t^2}{3} \ln t - \frac{4}{9}t^2$
25. $\frac{2}{(s+9)^3}$
26. $\frac{1}{s-2} - \frac{6}{s^4} + \frac{2}{s^3} - \frac{5}{s^2+25}$
27. $\frac{s^2-36}{(s^2+36)^2}$
28. $\frac{2}{s^2+1} + \frac{3s}{s^2+4}$
29. $\frac{6}{(s+5)^2+36}$
30. $\frac{2s^3-6sa^2}{(s^2+a^2)^2}$
31. $\frac{1-2e^{-2s}+e^{-4s}}{s}$
32. $\frac{2e^{-10s}}{s^3} + \frac{20e^{-10s}}{s^2}$
33. $\frac{1}{8}(e^{9t} - e^t)$
34. $2 \cos 3t - 6 \sin 3t$
35. $2 \cos 5t + \frac{18}{5} \sin 5t$

36. $\cos\sqrt{5}t + \frac{3}{\sqrt{5}}\sin\sqrt{5}t$

37. $e^{3t}\cos 4t$

38. $\frac{1}{4}(1-\cos 2t)$

39. $te^{-t} + 2e^{-t} + t - 2$

40. $1 - h(t-1)$

41. $\sin t\,(1 - h(t-\pi))$

42. (a) $sI - A = \begin{bmatrix} s-1 & 2 \\ 3 & s-2 \end{bmatrix}$; $(sI-A)^{-1} = \begin{bmatrix} \dfrac{s-2}{(s-4)(s+1)} & \dfrac{-2}{(s-4)(s+1)} \\ \dfrac{-3}{(s-4)(s+1)} & \dfrac{s-1}{(s-4)(s+1)} \end{bmatrix}$

(b) $\frac{1}{5}\begin{bmatrix} 2e^{4t}+3e^{-t} & -2e^{4t}+2e^{-t} \\ -3e^{4t}+3e^{-t} & -3e^{4t}+8e^{-t} \end{bmatrix}$ (c) e^{At} is same as $\mathcal{L}^{-1}((sI-A)^{-1})$.

(d) $\mathbf{y}(t) = \frac{1}{5}\begin{bmatrix} -8e^{4t}+3e^{-t} \\ 21e^{-t}-6e^{4t} \end{bmatrix}$

43. $\mathbf{y}(t) = \frac{1}{6}\begin{bmatrix} (5c_1-c_2)e^{4t} + (c_1+c_1)e^{-2t} \\ (-5c_1+c_2)e^{4t} + (5c_2+5c_1)e^{-2t} \end{bmatrix}$

44. $\mathbf{y}(t) = \frac{1}{2}\begin{bmatrix} 1+e^{4t} \\ -2+2e^{4t} \end{bmatrix}$

45. $\mathbf{y}(t) = \begin{bmatrix} e^{3t}+3te^{3t} \\ -2e^{3t}-3te^{3t} \end{bmatrix}$