Instructions. Answer each of the questions on your own paper. Put your name on each page of your paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* A copy of the table of Laplace transforms and convolution products from the text will be supplied, and these tables can be used for all problems.

1. [24 Points] Compute the Laplace transform of each of the following functions.

   (a) $f_1(t) = t^4 e^{-t/2}$

   (b) $f_2(t) = e^{2t}(3\cos 5t + 5\sin 3t)$

   (c) $f_3(t) = (2t+1)(t^2+3)$

   (d) $f_4(t) = e^{5t} * \sin \pi t = \int_0^t e^{5x} \sin \pi(t-x)\, dx$

2. [10 Points] Find the Laplace transform of the solution $y(t)$ of the initial value problem

$$2y'' - y' + 4y = 7\cos 3t, \qquad y(0) = -2,\ y'(0) = 4.$$

   Note that you are only asked to find the Laplace transform $Y(s)$ of $y(t)$, not $y(t)$ itself.

3. [24 Points] Compute the inverse Laplace transform of each of the following functions.

   (a) $F(s) = \dfrac{2s+3}{s^2+s-12}$

   (b) $G(s) = \dfrac{s^3+s^2+4}{s^2(s^2+2)}$

   (c) $H(s) = \dfrac{5s+12}{s^2+6s+13}$

4. [21 Points] Find the characteristic polynomial and the general solution of each of the following constant coefficient linear homogeneous differential equations:

   (a) $4y'' - 4y' + y = 0$

   (b) $4y'' + 4y' - 3y = 0$

   (c) $y'' + 4y' + 9y = 0$

5. [21 Points] Find the general solution of the constant coefficient homogeneous linear differential equation with the given characteristic polynomial $p(s)$.

   (a) $p(s) = (s^2-5)^2(s-5)^3$

   (b) $p(s) = s^4 + 3s^2 - 4$

   (c) $p(s)$ has degree 4 and has roots -2 with multiplicity 2 and $2 \pm 3i$, each with multiplicity 1.