Instructions. Answer each of the questions on your own paper. Put your name on each page of your paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* A copy of the Table of Laplace transforms from the text will be provided.

1. **[16 Points]** Solve: $2t^2y'' + 7ty' - 3y = 0$.

2. **[16 Points]** Solve: $y'' + y' - 6y = 3e^{2t}$.

3. **[20 Points]** Find the unique solution of the initial value problem

$$y'' + 2y' + 5y = 0, \quad y(0) = 3, \quad y'(0) = -1.$$

   Is the equation under damped or over damped? Does $y(t) = 0$ for some $t > 0$? If so, find the first $t > 0$ for which $y(t) = 0$. Sketch the graph of your solution.

4. **[16 Points]** Find the Laplace transform of the following function:

$$f(t) = \begin{cases} \sin 2t & \text{if } 0 \le t < \pi, \\ 0 & \text{if } t \ge \pi. \end{cases}$$

5. **[16 Points]** Find the inverse Laplace transform of the following function:

$$F(s) = \frac{(s+1)e^{-\pi s}}{s^2 + 2s + 10}$$

6. **[16 Points]** Solve the following initial value problem:

$$y'' + 5y' + 6y = \delta(t-4), \quad y(0) = 0,\ y'(0) = 0.$$

   Recall that $\delta(t-c)$ refers to the Dirac delta function which produces a unit impulse at time $t = c$. In the Table of Laplace transforms, this is referred to as $\delta_c(t)$ (that is $\delta_c(t) = \delta(t-c)$). (See Formula 24, Page 431 of the Laplace Transform Tables).