Instructions. Answer each of the questions on your own paper. Put your name on each page of your paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* As usual, a copy of the Table of Laplace transforms from the text will be provided.

In Exercises 1 – 7, solve the given differential equation. If initial values are given, solve the initial value problem. Otherwise, give the general solution. Some problems may be solvable by more than one technique. You are free to choose whatever technique that you deem to be most appropriate.

1. [**8 Points**] $(t^2+1)y' + 4ty = 4t, \quad y(0) = 2.$
2. [**8 Points**] $y'/x = e^y, \quad y(0) = 0.$
3. [**8 Points**] $y'' + 8y' + 12y = 0, \quad y(0) = 3,\ y'(0) = -14.$
4. [**8 Points**] $t^2y'' + ty' - 4y = 0.$
5. [**8 Points**] $y'' + 4y' + 9y = 0.$
6. [**8 Points**] $y'' + 7y' + 12y = \cos t.$
7. [**8 Points**] $y'' - 4y' + 13y = \delta(t), \quad y(0) = 0,\ y'(0) = 0.$ Recall that $\delta(t)$ refers to the Dirac delta function.
8. [**8 Points**] Find a particular solution of the differential equation

$$y'' - \frac{2}{t}y' + \frac{2}{t^2}y = t\ln t, \quad (t > 0)$$

given the fact that the general solution of the associated homogeneous equation is

$$y_h = c_1t + c_2t^2.$$

The integral formula

$$\int u^n \ln u\, du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$$

may be of use.

9. [**8 Points**] Find the Laplace transform of each of the following functions.

(a) $f(t) = \begin{cases} t^2 & \text{if } 0 \le t < 2, \\ 3 & \text{if } t \ge 2. \end{cases}$

(b) $g(t) = e^{2t}(t-1)^2 + e^{-2t}\cos 5t$

10. [**8 Points**] Compute each of the following inverse Laplace transforms.

    (a) $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+4s+5}\right\}$

    (b) $\mathcal{L}^{-1}\left\{\dfrac{e^{-3s}}{s(s+1)}\right\}$

11. [**12 Points**] Let $A = \begin{bmatrix} 5 & 3 \\ -3 & -1 \end{bmatrix}$.

    (a) Compute $(sI - A)^{-1}$.

    (b) Find $\mathcal{L}^{-1}\left\{(sI - A)^{-1}\right\}$.

    (c) Find the general solution of the system $\mathbf{y}' = A\mathbf{y}$.

    (d) Solve the initial value problem $\mathbf{y}' = A\mathbf{y}$, $\mathbf{y}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

12. [**8 Points**] A tank initially contains 60 gallons of pure water. Brine (a water-salt mixture) containing 1 pound of salt per gallon flows into the tank at the rate of 2 gal/min, and the mixture (which is assumed to be perfectly mixed) flows out of the tank at the rate of 3 gal/min; thus the tank is empty after exactly one hour.

    Set up an initial value problem that will enable you to calculate the amount of salt in the tank at any time until it is empty. **Do not solve the equation! Just set it up.**