Instructions. Answer each of the questions on your own paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* Put your name on each page of your paper. A short table of Laplace Transforms and a short table of integrals is included on Page 2.

1. [15 Points Each] Solve each of the following differential equations. If no initial condition is specified, give the general solution. If an initial condition is given, find the specific solution satisfying that initial condition. Be sure to show all of your work.

   (a) $y' = 2y + 4t$

   (b) $y' = \dfrac{2t}{t^2y + y}$

   (c) $2ty^2 + 1 + 2t^2yy' = 0$

   (d) $y' = y^2 - y, \quad y(0) = 1/3$

2. [10 Points] Apply Picard's method to compute the approximations $y_1(t)$, $y_2(t)$, and $y_3(t)$ to the solution of the initial value problem

$$y' = 2t + y, \quad y(0) = 0.$$

3. [15 Points] Compute the Laplace transform of each of the following functions.

   (a) $f(t) = 3e^{-7t} - 7t^3$

   (b) $g(t) = e^{-t/3}\cos\sqrt{6}t$

   (c) $h(t) = t^2e^{3t} + t^3e^{2t}$

4. [15 Points] A 1000 gallon tank is initially full of brine which contains 100 pounds of salt. A solution containing 4.0 pounds of salt per gallon enters the tank at a flow rate of 5 gallons per minute. A drain is opened at the bottom of the tank through which the well stirred solution leaves the tank at the same flow rate of 5 gallons per minute. Let $y(t)$ denote the amount of salt (in pounds) which is in the tank at time t.

   (a) What is $y(0)$? That is, how much salt is in the tank at time $t = 0$?

   (b) Find the amount $y(t)$ of salt in the tank for all times t.

   (c) How much salt is in the tank after 2 hours?

   (d) What is $\lim_{t\to\infty} y(t)$?

Exam I Supplementary Sheet

A Short Table of Laplace Transforms

1.	$\mathcal{L}\{af(t)+bg(t)\}(s)$	$=$	$aF(s)+bG(s)$
2.	$\mathcal{L}\{e^{at}f(t)\}(s)$	$=$	$F(s-a)$
3.	$\mathcal{L}\{1\}(s)$	$=$	$\dfrac{1}{s}$
4.	$\mathcal{L}\{t^n\}(s)$	$=$	$\dfrac{n!}{s^{n+1}}$
5.	$\mathcal{L}\{e^{at}\}(s)$	$=$	$\dfrac{1}{s-a}$
6.	$\mathcal{L}\{t^n e^{\alpha t}\}(s)$	$=$	$\dfrac{n!}{(s-\alpha)^{n+1}}$
7.	$\mathcal{L}\{\cos bt\}(s)$	$=$	$\dfrac{s}{s^2+b^2}$
8.	$\mathcal{L}\{\sin bt\}(s)$	$=$	$\dfrac{b}{s^2+b^2}$
9.	$\mathcal{L}\{e^{at}\cos bt\}(s)$	$=$	$\dfrac{s-a}{(s-a)^2+b^2}$
10.	$\mathcal{L}\{e^{at}\sin bt\}(s)$	$=$	$\dfrac{b}{(s-a)^2+b^2}$

Some Integral Formulas

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1}+C$ (if $n \neq -1$)
2. $\int \dfrac{1}{x}\,dx = \ln|x|+C$
3. $\int \dfrac{1}{a+bx}\,dx = \dfrac{1}{b}\ln|a+bx|+C \quad (b\neq 0)$
4. $\int \dfrac{1}{a^2+x^2}\,dx = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a}+C \quad (a>0)$
5. $\int \dfrac{1}{x(a+bx)}\,dx = \dfrac{1}{a}\ln\left|\dfrac{x}{a+bx}\right|+C$
6. $\int \dfrac{1}{a^2-x^2}\,dx = \dfrac{1}{2a}\ln\left|\dfrac{a+x}{a-x}\right|+C$
7. $\int \ln x\,dx = x\ln x - x + C$
8. $\int xe^{ax}\,dx = \dfrac{xe^{ax}}{a} - \dfrac{e^{ax}}{a^2}+C$