Instructions. Answer each of the questions on your own paper. Put your name on each page of your paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* A table of Laplace transforms has been appended to the exam. The following trigonometric identities may also be of use:

$$\boxed{\begin{array}{rcl} \sin(\theta+\varphi) & = & \sin\theta\cos\varphi + \sin\varphi\cos\theta \\ \cos(\theta+\varphi) & = & \cos\theta\cos\varphi - \sin\theta\sin\varphi \end{array}}$$

1. Solve: [**12 Points**] $t^2y'' - 2ty' - 10y = 0$.

2. Solve: [**16 Points**] $y'' - y' - 2y = 2e^{-t}$.

3. [**20 Points**] You may assume that $\mathcal{S} = \{e^{2t},\, te^{2t}\}$ is a fundamental set of solutions for the homogeneous equation
$$y'' - 4y' + 4y = 0.$$
Use *variation of parameters* to find a particular solution of the nonhomogeneous differential equation
$$y'' - 4y' + 4y = t^{5/2}e^{2t}.$$

4. [**14 Points**] Find the Laplace transform of the following function:
$$f(t) = \begin{cases} 2 & \text{if } 0 \le t < 2, \\ 6 - 2t & \text{if } 2 \le t < 4 \\ -2 & \text{if } t \ge 4. \end{cases}$$

5. [**16 Points**] Find the inverse Laplace transform of the following functions:
   (a) $F(s) = \dfrac{2}{(s+4)^3}e^{-3s}$
   (b) $G(s) = \dfrac{2s}{s^2+4}e^{-3s}$

6. [**22 Points**] Solve the following initial value problem:
$$y'' + y = \delta_{\pi/2}(t) \quad y(0) = 1,\ y'(0) = 0.$$
Give a careful sketch of the graph of the solution for the interval $0 \le t \le 2\pi$.
Recall that δ_c is the Dirac delta function which produces a unit impulse at time $t = c$.

A Short Table of Laplace Transforms

1.	$\mathcal{L}\{af(t)+bg(t)\}(s)$	$=$	$aF(s)+bG(s)$
2.	$\mathcal{L}\{e^{at}f(t)\}(s)$	$=$	$F(s-a)$
3.	$\mathcal{L}\{-tf(t)\}(s)$	$=$	$\frac{d}{ds}F(s)$
4.	$\mathcal{L}\{1\}(s)$	$=$	$\frac{1}{s}$
5.	$\mathcal{L}\{t^n\}(s)$	$=$	$\frac{n!}{s^{n+1}}$
6.	$\mathcal{L}\{e^{at}\}(s)$	$=$	$\frac{1}{s-a}$
7.	$\mathcal{L}\{t^n e^{\alpha t}\}(s)$	$=$	$\frac{n!}{(s-\alpha)^{n+1}}$
8.	$\mathcal{L}\{\cos bt\}(s)$	$=$	$\frac{s}{s^2+b^2}$
9.	$\mathcal{L}\{\sin bt\}(s)$	$=$	$\frac{b}{s^2+b^2}$
10.	$\mathcal{L}\{e^{at}\cos bt\}(s)$	$=$	$\frac{s-a}{(s-a)^2+b^2}$
11.	$\mathcal{L}\{e^{at}\sin bt\}(s)$	$=$	$\frac{b}{(s-a)^2+b^2}$
12.	$\mathcal{L}\{f'(t)\}(s)$	$=$	$sF(s)-f(0)$
13.	$\mathcal{L}\{f''(t)\}(s)$	$=$	$s^2F(s)-sf(0)-f'(0)$
14.	$\mathcal{L}\{(f*g)(t)\}(s)$	$=$	$F(s)G(s)$
15.	$\mathcal{L}\{h(t-c)\}(s)$	$=$	$\frac{e^{-sc}}{s}$
16.	$\mathcal{L}\{f(t-c)h(t-c)\}(s)$	$=$	$e^{-sc}F(s)$
16'.	$\mathcal{L}\{g(t)h(t-c)\}(s)$	$=$	$e^{-sc}\mathcal{L}\{g(t+c)\}$
17.	$\mathcal{L}\{\delta_c(t)\}(s)$	$=$	e^{-sc}