Instructions. Answer each of the questions on your own paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* Put your name on each page of your paper. A table of Laplace transforms and a short table of integrals are appended to the exam.

1. **[17 Points]** Find the general solution of: $y' = 6t(y-1)^{2/3}$.

2. **[17 Points]** Find the general solution of: $y' - 4y = 3e^{4t} + 4e^{3t}$.

3. **[17 Points]** Solve the initial value problem: $y' + \dfrac{6}{t}y = 11t^4, \quad y(1) = 3.$

4. **[17 Points]** Find the general solution of: $y\cos t + (\sin t + 3y^2)y' = 0$.

5. **[4 Points]** Complete the following definition: Suppose $f(t)$ is a continuous function defined for all $t \geq 0$. The **Laplace transform** of f is the function $F(s)$ defined as follows:

$$F(s) = \mathcal{L}\{f(t)\}(s) = \boxed{}$$

   for all s sufficiently large.

6. **[12 Points]** Compute the Laplace transform of each of the following functions. You may use the attached tables, but be sure to identify which formulas you are using by citing the number(s) in the table.

   (a) $f(t) = 4t^3 - 5t^2 + 7$

   (b) $g(t) = 2e^{3t} + t^2e^{-2t} + 5e^t\cos 2t$

7. **[16 Points]** A tank contains 100 gallons of pure water. A solution containing 0.5 pounds of salt per gallon enters the tank at a flow rate of 3 gallons per minute. A drain is opened at the bottom of the tank through which the well stirred solution leaves the tank at the same flow rate of 3 gallons per minute. Let $y(t)$ denote the amount of salt (in pounds) which is in the tank at time t.

   (a) What is $y(0)$? That is, how much salt is in the tank at time $t = 0$?

   (b) Find the amount $y(t)$ of salt in the tank for all times t.

   (c) How much salt is in the tank after 1 hour?

   (d) What is $\lim_{t\to\infty} y(t)$?

Laplace Transform Tables

	$f(t)$	$\to$	$F(s) = \mathcal{L}\{f(t)\}(s)$
1.	1	$\to$	$\frac{1}{s}$
2.	t^n	$\to$	$\frac{n!}{s^{n+1}}$
3.	e^{at}	$\to$	$\frac{1}{s-a}$
4.	$t^n e^{at}$	$\to$	$\frac{n!}{(s-a)^{n+1}}$
5.	$\cos bt$	$\to$	$\frac{s}{s^2+b^2}$
6.	$\sin bt$	$\to$	$\frac{b}{s^2+b^2}$
7.	$e^{at}\cos bt$	$\to$	$\frac{s-a}{(s-a)^2+b^2}$
8.	$e^{at}\sin bt$	$\to$	$\frac{b}{(s-a)^2+b^2}$

Linearity	$\mathcal{L}\{af(t)+bg(t)\}$	$=$	$a\mathcal{L}\{f\}+b\mathcal{L}\{g\}$
Input Derivative Principles	$\mathcal{L}\{f'(t)\}(s)$	$=$	$s\mathcal{L}\{f(t)\}-f(0)$
	$\mathcal{L}\{f''(t)\}(s)$	$=$	$s^2\mathcal{L}\{f(t)\}-sf(0)-f'(0)$
First Translation Principle	$\mathcal{L}\{e^{at}f(t)\}$	$=$	$F(s-a)$
Transform Derivative Principle	$\mathcal{L}\{-tf(t)\}(s)$	$=$	$\frac{d}{ds}F(s)$
The Dilation Principle	$\mathcal{L}\{f(bt)\}(s)$	$=$	$\frac{1}{b}\mathcal{L}\{f(t)\}(s/b).$

Some Integral Formulas

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$ (if $n \neq -1$)
2. $\int \frac{1}{x}\,dx = \ln|x| + C$
3. $\int \frac{1}{a+bx}\,dx = \frac{1}{b}\ln|a+bx| + C \quad (b \neq 0)$
4. $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \quad (a > 0)$
5. $\int \frac{1}{x(a+bx)}\,dx = \frac{1}{a}\ln\left|\frac{x}{a+bx}\right| + C$
6. $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$
7. $\int \ln x\,dx = x\ln x - x + C$
8. $\int xe^{ax}\,dx = \frac{xe^{ax}}{a} - \frac{e^{ax}}{a^2} + C$
9. $\int \cos ax\,dx = \frac{\sin ax}{a} + C$
10. $\int \sin ax\,dx = -\frac{\cos ax}{a} + C$