Math 2065 Section 1
Review Exercises for Exam I

The syllabus for Exam I is Chapter 1 and Section 2.1. A copy of Tables 2.1 and 2.2 on Pages 103 and 104 will be provided with the exam.

You should bring your own paper for the exam, since space will (in general) not be provided on the exam paper for answers. You will need to show all relevant work in order to receive partial credit.

1. Determine if each of the following equations is separable (S), linear (L), homogeneous (H), Bernoulli(B), exact (E), or none of these (N). List all types that apply for each equation. Some algebraic manipulation may be needed to put the equation in the standard form for determining each type. Do **not** solve the equations!

   (a) $t^2 y' = 1 - 2ty$

   (b) $yy' = 3 - 2y$

   (c) $y' = y^2 - ty$

   (d) $y' = y^2 - t$

   (e) $ty' = y - 2ty$

   (f) $(t^2 + 3y^2)y' = -2ty$

   (g) $(2ty - y^3)y' = -1 - t - y^2$

   (h) $t + y' = y - 2ty$

2. Find the general solution of each of the following differential equations. You **must** show your work.

   (a) $y' + 2y = 0$

   (b) $y' + 2y = 3e^t$

   (c) $y' - y = e^{3t}$

   (d) $y' + 2ty = t$

   (e) $y' + \frac{3}{t}y = \frac{\sin t}{t^3}$

   (f) $y' = \frac{t^2}{y}$

   (g) $ty' = \sqrt{1 - y^2}$

   (h) $y' = y^2 + 4$

   (i) $2ty^2 - 3 + (2t^2 y + 4)y' = 0$

   (j) $ty' = 2te^t - y + 6t^2$

(k) $y' + y = e^t y^2$

3. Solve each of the following initial value problems. You **must** show your work.

   (a) $y' = 2y + 5e^{2t}, \quad y(0) = -1.$
   (b) $y' = y^2 t^3, \quad y(1) = -1.$
   (c) $y' + 3y = 4e^{-3t} \sin 2t, \quad y(0) = -1.$
   (d) $y' + \dfrac{3}{t} y = 7t^3, \quad y(1) = -1.$
   (e) $(4y + 2t - 5) + (6y + 4t - 1)y' = 0, \quad y(-1) = 2.$

4. Newton's law of cooling states that the rate at which a body cools (or heats up) is proportional to the difference between the temperature of the body and the temperature of the surrounding medium. A turkey which is initially at room temperature (70° F) is placed in a 350° F oven at time $t = 0$. Write an initial value problem which is satisfied by the temperature $T(t)$ of the turkey at time t.

5. A tank contains 300 gal of brine made by dissolving 50 lb of salt in water. A salt solution containing 2 lb per gallon of water runs into the tank at the rate of 3 gal/min, and the mixture, which is kept uniform by stirring, runs out at the same rate.

   (a) Find the amount $y(t)$ of salt in the tank at any time t.
   (b) What is $\lim_{t\to\infty} y(t)$? Does your answer make sense?

6. Apply Picard's method to compute the first two approximations $y_1(t)$ and $y_2(t)$ to the solution $y(t)$ of the initial value problem

$$y' = t^2 + y^2, \quad y(0) = 0.$$

7. (a) Complete the following definition: Suppose $f(t)$ is a continuous function of exponential type defined for all $t \geq 0$. The **Laplace transform** of f is the function $F(s)$ defined as follows

$$F(s) = \mathcal{L}(f(t))(s) = \boxed{}$$

   for all s sufficiently large.

   (b) Using your definition compute the Laplace transform of the function $f(t) = 2t - 5$. You may need to review the integration by parts formula: $\int u\,dv = uv - \int v\,du$.

8. Compute the Laplace transform of each of the following functions using the Laplace transform Tables. (See Pages 103–104.)

(a) $f(t) = 3t^3 - 2t^2 + 7$

(b) $g(t) = e^{-3t} + \sin \sqrt{2}t$

(c) $h(t) = -8 + \cos(t/2)$

(d) $f(t) = 7e^{2t} \cos 3t - 2e^{7t} \sin 5t$

(e) $g(t) = 3t \sin 2t$

(f) $h(t) = (2 - t^2)e^{-5t}$

(g) $t^2 e^{-9t}$

(h) $e^{2t} - t^3 + t^2 - \sin 5t$

(i) $t \cos 6t$

(j) $2 \sin t + 3 \cos 2t$

(k) $e^{-5t} \sin 6t$

(l) $t^2 \cos at$ where a is a constant

(m) $f(t) = 3(e^t)^4 + \sin \sqrt{2}t$

(n) $g(t) = 5t^3 - 3 \cos 5t + \frac{3}{5}$