Instructions. Answer each of the questions on your own paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* Put your name on each page of your paper. A table of Laplace transforms and a short table of integrals are appended to the exam.

1. [**17 Points**] Solve the initial value problem: $y' = \dfrac{3t^2}{2(y-1)}, \quad y(0) = 0.$

2. [**17 Points**] Solve the initial value problem: $y' + 2y = 3e^{2t} + 4e^{-2t} \quad y(0) = -1.$

3. [**17 Points**] Solve the initial value problem: $y' + \dfrac{2t}{1+t^2}y = 1, \quad y(0) = 0.$

4. [**17 Points**] Find the general solution of: $2ty^2 - 3 + (2t^2y + 4)y' = 0.$

5. [**15 Points**] Compute the Laplace transform of each of the following functions. You may use the attached tables, but be sure to identify which formulas you are using by citing the number(s) or name of the formula in the table.

   (a) $f(t) = 3t^4 - 5te^{-6t} + 7$

   (b) $g(t) = (t-3)^2e^{4t} + 5e^{-2t}\cos 3t$

   (c) $h(t) = 3t\sin 5t$

6. [**17 Points**] A 400 gallon tank is initially full of brine which contains 60 pounds of salt. A solution containing 0.5 pounds of salt per gallon enters the tank at a rate of 6 gallons per minute. A drain is opened at the bottom of the tank through which the well stirred solution leaves the tank at the same rate of 6 gallons per minute. Let $y(t)$ denote the amount of salt (in pounds) which is in the tank at time t.

   (a) What is $y(0)$?

   (b) Write the differential equation that $y(t)$ must satisfy.

   (c) Solve the differential equation to find $y(t)$.

   (d) How much salt is in the tank after 1 hour?

Laplace Transform Tables

	$f(t)$	$\to$	$F(s) = \mathcal{L}\left\{f(t)\right\}(s)$
1.	1	$\to$	$\dfrac{1}{s}$
2.	t^n	$\to$	$\dfrac{n!}{s^{n+1}}$
3.	e^{at}	$\to$	$\dfrac{1}{s-a}$
4.	$t^n e^{at}$	$\to$	$\dfrac{n!}{(s-a)^{n+1}}$
5.	$\cos bt$	$\to$	$\dfrac{s}{s^2+b^2}$
6.	$\sin bt$	$\to$	$\dfrac{b}{s^2+b^2}$
7.	$e^{at}\cos bt$	$\to$	$\dfrac{s-a}{(s-a)^2+b^2}$
8.	$e^{at}\sin bt$	$\to$	$\dfrac{b}{(s-a)^2+b^2}$

Linearity	$\mathcal{L}\left\{af(t)+bg(t)\right\}$	$=$	$a\mathcal{L}\left\{f\right\}+b\mathcal{L}\left\{g\right\}$
Input Derivative Principles	$\mathcal{L}\left\{f'(t)\right\}(s)$	$=$	$s\mathcal{L}\left\{f(t)\right\}-f(0)$
	$\mathcal{L}\left\{f''(t)\right\}(s)$	$=$	$s^2\mathcal{L}\left\{f(t)\right\}-sf(0)-f'(0)$
First Translation Principle	$\mathcal{L}\left\{e^{at}f(t)\right\}$	$=$	$F(s-a)$
Transform Derivative Principle	$\mathcal{L}\left\{-tf(t)\right\}(s)$	$=$	$\dfrac{d}{ds}F(s)$
The Dilation Principle	$\mathcal{L}\left\{f(bt)\right\}(s)$	$=$	$\dfrac{1}{b}\mathcal{L}\left\{f(t)\right\}(s/b).$

Some Integral Formulas

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$ (if $n \neq -1$)
2. $\int \frac{1}{x}\,dx = \ln|x| + C$
3. $\int \frac{1}{a+bx}\,dx = \frac{1}{b}\ln|a+bx| + C \quad (b \neq 0)$
4. $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \quad (a > 0)$
5. $\int \frac{1}{x(a+bx)}\,dx = \frac{1}{a}\ln\left|\frac{x}{a+bx}\right| + C$
6. $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$
7. $\int \ln x\,dx = x\ln x - x + C$
8. $\int xe^{ax}\,dx = \frac{xe^{ax}}{a} - \frac{e^{ax}}{a^2} + C$
9. $\int \cos ax\,dx = \frac{\sin ax}{a} + C$
10. $\int \sin ax\,dx = -\frac{\cos ax}{a} + C$