Instructions. Answer each of the questions on your own paper. Put your name on each page of your paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* A table of Laplace transforms has been appended to the exam.

1. [**15 Points**] Each of the following matrices is in reduced row echelon form:

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

   For each of these matrices, write down all solutions to the linear system of equations that has the given matrix as augmented matrix.

2. [**20 Points**] Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -3 & 3 \\ 1 & -1 & 1 \end{bmatrix}$.

   (a) Compute A^{-1}.

   (b) Using your answer to part (a), solve the linear system $A\mathbf{x} = \mathbf{b}$ if $\mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$.

3. [**15 Points**] Let $C = \begin{bmatrix} 0 & 0 & 2 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & -1 & 6 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}$. Compute $\det C$.

4. [**15 Points**] If $A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$, compute the matrix exponential e^{At}.

5. [**20 Points**] Find the general solution of the system of differential equations

$$\begin{aligned} y_1' &= 3y_1 - 6y_2 \\ y_2' &= 3y_1 - 3y_2 \end{aligned}$$

   You may assume that the characteristic polynomial of the matrix $A = \begin{bmatrix} 3 & -6 \\ 3 & -3 \end{bmatrix}$ is $c_A(s) = s^2 + 9$.

6. [**20 Points**] If $A = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$, solve the initial value problem $\mathbf{y}' = A\mathbf{y}$, $\mathbf{y}(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$. You may assume that the characteristic polynomial of A is $c_A(s) = (s-3)^2$.

Laplace Transform Table

	$f(t)$	$\to$	$F(s) = \mathcal{L}\{f(t)\}(s)$
1.	1	$\to$	$\dfrac{1}{s}$
2.	t^n	$\to$	$\dfrac{n!}{s^{n+1}}$
3.	e^{at}	$\to$	$\dfrac{1}{s-a}$
4.	$t^n e^{at}$	$\to$	$\dfrac{n!}{(s-a)^{n+1}}$
5.	$\cos bt$	$\to$	$\dfrac{s}{s^2+b^2}$
6.	$\sin bt$	$\to$	$\dfrac{b}{s^2+b^2}$
7.	$e^{at}\cos bt$	$\to$	$\dfrac{s-a}{(s-a)^2+b^2}$
8.	$e^{at}\sin bt$	$\to$	$\dfrac{b}{(s-a)^2+b^2}$
9.	$h(t-c)$	$\to$	$\dfrac{e^{-sc}}{s}$
10.	$\delta_c(t) = \delta(t-c)$	$\to$	e^{-sc}

Laplace Transform Principles

Linearity	$\mathcal{L}\{af(t)+bg(t)\}$	$=$	$a\mathcal{L}\{f\}+b\mathcal{L}\{g\}$
Input Derivative Principles	$\mathcal{L}\{f'(t)\}(s)$	$=$	$s\mathcal{L}\{f(t)\}-f(0)$
	$\mathcal{L}\{f''(t)\}(s)$	$=$	$s^2\mathcal{L}\{f(t)\}-sf(0)-f'(0)$
First Translation Principle	$\mathcal{L}\{e^{at}f(t)\}$	$=$	$F(s-a)$
Transform Derivative Principle	$\mathcal{L}\{-tf(t)\}(s)$	$=$	$\dfrac{d}{ds}F(s)$
Second Translation Principle	$\mathcal{L}\{h(t-c)f(t-c)\}$	$=$	$e^{-sc}F(s)$, or
	$\mathcal{L}\{g(t)h(t-c)\}$	$=$	$e^{-sc}\mathcal{L}\{g(t+c)\}$.
The Convolution Principle	$\mathcal{L}\{(f*g)(t)\}(s)$	$=$	$F(s)G(s)$.