

**Instructions.** Answer each of the questions on your own paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* Put your name on each page of your paper.

1. [15 Points] Each of the following matrices is in reduced row echelon form:

$$A = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -4 & 0 & 2 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 5 & 0 & -3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For each of these matrices, write down all solutions to the linear system of equations that has the given matrix as augmented matrix.

2. [15 Points] Let  $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{bmatrix}$ .

(a) Compute  $A^{-1}$ .

(b) Using your answer to part (a), solve the linear system  $A\mathbf{x} = \mathbf{b}$  if  $\mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ .

3. [10 Points] Let  $C = \begin{bmatrix} 0 & 0 & 5 & 2 \\ 2 & 4 & 6 & 2 \\ 1 & -1 & 6 & 0 \\ 0 & 0 & -3 & 2 \end{bmatrix}$ . Compute  $\det C$ .

4. [20 Points] Solve the initial value problem:  $y' + 3y = 12e^{3t} - 3e^{-3t}$ ,  $y(0) = -3$ .

5. [20 Points] Solve the initial value problem:  $ty' + 6y = 11t^5$ ,  $y(1) = 3$ .

6. [20 Points] Solve the initial value problem:  $(1 + t^2)y' = -2ty^2$ ,  $y(0) = 1$ .