

Instructions. Answer each of the questions on your own paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* Put your name on each page of your paper.

1. [8 Points] Short Answer. For True/False questions you must write ‘True’ or ‘False’ and justify your answer.
 - (a) True or False: The differential equation $y' = e^{t^2 + \sin(y)}$ is separable.
 - (b) True or False: The differential equation $y' + y^2t = t^2$ is first order linear.
 - (c) What are the possible number of solutions to a system of linear equations?
 - (d) Let A be a 2015×2015 matrix. If the equation $A\mathbf{x} = \mathbf{0}$ has only the solution $\mathbf{x} = \mathbf{0}$, what do we know about $\det A$?
2. [16 Points] Find all solutions to the linear system

$$\begin{array}{rccccrcr} x_1 & + & x_2 & + & x_3 & + & x_4 & = & 12 \\ & & & & x_2 & - & x_3 & + & 4x_4 & = & 5 \\ 3x_1 & + & 2x_2 & + & 4x_3 & - & x_4 & = & 31 \end{array}$$

3. [16 Points] Let $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$.

(a) Compute the inverse of A .

(b) Using your answer to part (a), solve the linear system $A\mathbf{x} = \mathbf{b}$ if $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$.

4. [12 Points]

(a) If A is a 3×3 matrix with $\det A = 3$ and B is obtained from A by first interchanging rows 1 and 2 and then multiplying row 3 by 5, what is $\det(2A^2B)$?

(b) Find all values of x for which C^{-1} fails to exist, where $C = \begin{bmatrix} 1 & 2 & 5x \\ 2x - 1 & 0 & 0 \\ 3x - 5 & 2 & 10 \end{bmatrix}$.

5. [16 Points] Solve the initial value problem: $y' = e^{2t}y^2$, $y(0) = -1$.

6. [16 Points] Solve the initial value problem: $y' + 4y = 6e^{-4t} + 4e^{-6t}$, $y(0) = 5$.

7. [16 Points] Find the general solution to the differential equation: $y' - 4ty = 2t$.