Instructions. Do each problem showing your work. Answers alone are not sufficient. Label each problem clearly, and write neatly, in a logical sequence.

Do the following problems from the text:
Page 109: 8, 12
Page 125: 12, 14, 18, 20, 26, 30

Page 109:

8. Apply the Laplace transform to both sides. For the left hand side we get

$$\begin{aligned}\mathcal{L}\{y''+5y'+6y\}(s) &= \mathcal{L}\{y''\}(s)+5\mathcal{L}\{y'\}(s)+6\mathcal{L}\{y\}(s)\\ &= s^2Y(s)-sy(0)-y'(0)+5(sY(s)-y(0))+6Y(s)\\ &= (s^2+5s+6)Y(s)-2s-4.\end{aligned}$$

Since the Laplace transform of 0 is 0, we now get

$$(s^2+5s+6)Y(s)-2s-4=0.$$

Hence,

$$Y(s)=\frac{2s+4}{s^2+5s+6}=\frac{2(s+2)}{(s+3)(s+2)}=\frac{2}{s+3},$$

and therefore, $y(t)=2e^{-3t}$.

12. Apply the Laplace transform to both sides. For the left hand side we get

$$\begin{aligned}\mathcal{L}\{y''-4y'+4y\}(s) &= \mathcal{L}\{y''\}(s)-4\mathcal{L}\{y'\}(s)+4\mathcal{L}\{y\}(s)\\ &= s^2Y(s)-sy(0)-y'(0)-4(sY(s)-y(0))+4Y(s)\\ &= (s^2-4s+4)Y(s)+s.\end{aligned}$$

Since $\mathcal{L}\{4e^{2t}\}(s)=4/(s-2)$ we get the algebraic equation

$$(s-2)^2Y(s)+s=\frac{4}{s-2}.$$

Hence,

$$\begin{aligned}Y(s) &= \frac{-s}{(s-2)^2}+\frac{4}{(s-2)^3}\\ &= \frac{-(s-2)-2}{(s-2)^2}+\frac{4}{(s-2)^3}\\ &= -\frac{1}{s-2}-\frac{2}{(s-2)^2}+2\frac{2}{(s-2)^3}\end{aligned}$$

and therefore $y(t)=-e^{2t}-2te^{2t}+2t^2e^{2t}$.

Page 125:

12. $\mathcal{L}\left\{e^t(t-\cos 4t)\right\}(s) = \mathcal{L}\left\{te^{-t}\right\}(s) - \mathcal{L}\left\{e^{-t}\cos 4t\right\}(s) = \dfrac{1}{(s-1)^2} - \dfrac{s-1}{(s-1)^2+16}.$

14. $\mathcal{L}\left\{e^{-t/3}\cos\sqrt{6}t\right\}(s) = \dfrac{s+\frac{1}{3}}{(s+\frac{1}{3})^2+6}$

18. $\mathcal{L}\left\{e^{5t}(8\cos 2t + 11\sin 2t)\right\}(s) = 8\mathcal{L}\left\{e^{5t}\cos 2t\right\}(s) + 11\mathcal{L}\left\{e^{5t}\sin 2t\right\}(s) = \dfrac{8(s-5)}{(s-5)^2+4} + \dfrac{22}{(s-5)^2+4} = \dfrac{8s-18}{s^2-10s+29}$

20. $\mathcal{L}\left\{t\cos 3t\right\}(s) = -\left(\mathcal{L}\left\{\cos 3t\right\}\right)' = -\left(\dfrac{s}{s^2+9}\right)' = \dfrac{s^2-9}{(s^2+9)^2}$

26. $\mathcal{L}\left\{\cos^2 bt\right\}(s) = \dfrac{1}{2}\mathcal{L}\left\{1+\cos 2bt\right\}(s) = \dfrac{1}{2}\left(\dfrac{1}{s} + \dfrac{s}{s^2+4b^2}\right) = \dfrac{s^2+2b^2}{s(s^2+4b^2)}$

30. $\mathcal{L}\left\{\cosh bt\right\} = \dfrac{1}{2}\left(\mathcal{L}\left\{e^{bt}+e^{-bt}\right\}\right) = \dfrac{1}{2}\left(\dfrac{1}{s+b} + \dfrac{1}{s-b}\right) = \dfrac{s}{s^2-b^2}$