Instructions. Do each problem showing your work. Answers alone are not sufficient. Label each problem clearly, and write neatly, in a logical sequence.

Do the following problems from the text:
Page 139: 12, 24, 28
Page 149: 8, 12
Page 163: 4, 8, 10, 16, 22, 24, 28

Page 139:

12. Use the technique of distinct linear factors (Page 136).

$$\frac{5s+9}{(s-1)(s+3)} = \frac{A_1}{s-1} + \frac{A_2}{s+3}$$

where $A_1 = \left.\frac{5s+9}{s+3}\right|_{s=1} = 14/4 = 7/2$ and $A_2 = \left.\frac{5s+9}{s-1}\right|_{s=-3} = -6/(-4) = 3/2$.
Thus

$$\frac{5s+9}{(s-1)(s+3)} = \frac{3/2}{s+3} + \frac{7/2}{s-1}$$

24. Use Theorem 1 Page 129 to compute the $(s+2)$-chain:

$$\frac{5s^2-3s+10}{(s+1)(s+2)^2} = \frac{A_1}{(s+2)^2} + \frac{p_1(s)}{(s+2)(s+1)}$$

$$\text{where} \quad A_1 = \left.\frac{5s^2-3s+10}{s+1}\right|_{s=-2} = -36$$

$$\text{and} \quad p_1(s) = \frac{1}{s+2}(5s^2-3s+10-(-36)(s+1)) = \frac{1}{s+2}(5s^2+33s+46) = 5s+23$$

Continuing gives

$$\frac{5s+23}{(s+2)(s+1)} = \frac{A_2}{s+2} + \frac{p_2(s)}{s+1}$$

$$\text{where} \quad A_2 = \left.\frac{5s+23}{s+1}\right|_{s=-2} = -13$$

$$\text{and} \quad p_2(s) = \frac{1}{s+2}(5s+23-(-13)(s+1)) = \frac{1}{s+2}(18s+36) = 18$$

Thus

$$\frac{5s^2-3s+10}{(s+1)(s+2)^2} = \frac{-36}{(s+2)^2} - \frac{13}{s+2} + \frac{18}{s+1}.$$

28. Use Theorem 1 Page 129 to compute the $(s+2)$-chain:

$$\frac{s^2}{(s+2)^2(s+1)^2} = \frac{A_1}{(s+2)^2} + \frac{p_1(s)}{(s+2)(s+1)^2}$$

$$\begin{aligned} \text{where} \quad A_1 &= \left.\frac{s^2}{(s+1)^2}\right|_{s=-2} = 4 \\ \text{and} \quad p_1(s) &= \frac{1}{s+2}(s^2-(4)(s+1)^2) \\ &= \frac{-3s^2-8s-4}{s+2} = \frac{-(3s+2)(s+2)}{s+2} = -(3s+2) \end{aligned}$$

Continuing gives

$$\frac{-3s-2}{(s+2)(s+1)^2} = \frac{A_2}{s+2} + \frac{p_2(s)}{(s+1)^2}$$

$$\begin{aligned} \text{where} \quad A_2 &= \left.\frac{-3s-2}{(s+1)^2}\right|_{s=-2} = 4 \\ \text{and} \quad p_2(s) &= \frac{1}{s+2}(-3s-2-(4)(s+1)^2) = -(4s+3) \end{aligned}$$

Thus $\frac{s^2}{(s+2)^2(s+1)^2} = \frac{4}{(s+2)^2} + \frac{4}{s+2} + \frac{-4s-3}{(s+1)^2}$. Now continue using Theorem 1 Page 129 or replace s by $(s+1)-1$ in the numerator of the last fraction to get $\frac{s^2}{(s+2)^2(s+1)^2} = \frac{4}{(s+2)^2} + \frac{4}{s+2} + \frac{1}{(s+1)^2} - \frac{4}{s+1}$

Page 149:

8. Use Theorem 1, Page 129, to compute the $(s+1)$-chain:

$$\frac{4s}{(s^2+1)^2(s+1)} = \frac{A_1}{s+1} + \frac{p_1(s)}{(s^2+1)^2}$$

$$\begin{aligned} \text{where} \quad A_1 &= \left.\frac{4s}{(s^2+1)^2}\right|_{s=-1} = -1 \\ \text{and} \quad p_1(s) &= \frac{1}{s+1}(4s-(-1)(s^2+1)^2) = \frac{s^4+2s^2+4s+1}{s+1} \\ &= s^3-s^2+3s+1 \end{aligned}$$

Now compute the s^2+1-chain for the remainder term $\frac{s^3-s^2+3s+1}{(s^2+1)^2}$ using Theorem 1 Page 143. Since $s=i$ is a root of s^2+1, an application of this theorem gives

$$\begin{aligned}
\frac{s^3 - s^2 + 3s + 1}{(s^2+1)^2} &= \frac{B_1 s + C_1}{(s^2+1)^2} + \frac{p_1(s)}{(s^2+1)} \\
\text{where} \quad B_1 i + C_1 &= \left.\frac{s^3 - s^2 + 3s + 1}{1}\right|_{s=i} = 2i + 2 \\
&\Rightarrow B_1 = 2 \quad \text{and } C_1 = 2 \\
\text{and} \quad p_1(s) &= \frac{1}{s^2+1}(s^3 - s^2 + 3s + 1 - (2s+2)(1)) \\
&= \frac{s^3 - s^2 + s - 1}{s^2+1} = s - 1.
\end{aligned}$$

Since the remainder term $\frac{s-1}{s^2+1}$ is a simple partial fraction, we conclude that the complete partial fraction decomposition is

$$\frac{4s}{(s^2+1)^2(s+1)} = \frac{2s+2}{(s^2+1)^2} + \frac{s-1}{s^2+1} - \frac{1}{s+1}$$

12. Use Theorem 1, Page 129 to compute the $(s-1)$-chain:

$$\frac{30}{(s^2-4s+13)(s-1)} = \frac{A_1}{s-1} + \frac{p_1(s)}{(s^2-4s+13)}$$

$$\begin{aligned}
\text{where} \quad A_1 &= \left.\frac{30}{(s^2-4s+13)}\right|_{s=1} = 3 \\
\text{and} \quad p_1(s) &= \frac{1}{s-1}(30 - (3)(s^2 - 4s + 13)) = \frac{-3s^2 + 12s - 9}{s-1} \\
&= -3s + 9
\end{aligned}$$

Since the remainder term $\frac{-3s+9}{s^2-4s+13}$ is a simple partial fraction, we conclude that the complete partial fraction decomposition is

$$\frac{30}{(s^2-4s+13)(s-1)} = \frac{9-3s}{((s-2)^2+9)} + \frac{3}{s-1}$$

Page 163:

4. $\mathcal{L}^{-1}\left\{\frac{4}{2s+3}\right\} = 2\mathcal{L}^{-1}\left\{\frac{1}{s+(3/2)}\right\} = 2e^{-3t/2}$

8. Partial fractions gives $\frac{2s-5}{(s+3)^3} = \frac{-11}{(s+3)^3} + \frac{2}{(s+3)^2}$. Thus $\mathcal{L}^{-1}\left\{\frac{2s-5}{(s+3)^3}\right\} = \frac{-11}{2}t^2e^{-3t} + 2te^{-3t}$.

10. $\dfrac{s}{s^2-5s+6} = \dfrac{s}{(s-2)(s-3)} = \dfrac{-2}{s-2} + \dfrac{3}{s-3}$. So

$$\mathcal{L}^{-1}\left\{\frac{s}{(s-2)(s+3)}\right\} = 3e^{3t} - 2e^{2t}.$$

16. $\dfrac{5s+15}{(s^2+9)(s-1)} = \dfrac{2}{s-1} - 2\dfrac{s}{s^2+9} + \dfrac{3}{s^2+9}$. So

$$\mathcal{L}^{-1}\left\{\frac{5s+15}{(s^2+9)(s-1)}\right\} = -2\cos 3t + \sin 3t + 2e^{t}.$$

22. $\dfrac{2s+4}{s^2-4s+12} = 2\dfrac{s-2}{(s-2)^2+(\sqrt{8})^2} + \sqrt{8}\dfrac{\sqrt{8}}{(s-2)^2+(\sqrt{8})^2}$. Thus

$$\mathcal{L}^{-1}\left\{\frac{2s+4}{s^2-4s+12}\right\} = 2e^{2t}\cos\sqrt{8}\,t + \sqrt{8}\,e^{2t}\sin\sqrt{8}\,t.$$

24. $\dfrac{s-5}{s^2-6s+13} = \dfrac{s-3}{(s-3)^2+2^2} - \dfrac{2}{(s-3)^2+2^2}$. Thus $\mathcal{L}^{-1}\left\{\dfrac{s-5}{s^2-6s+13}\right\} = e^{3t}\cos 2t - e^{3t}\sin 2t$.

28. We first complete the square $s^2-6s+10 = (s-3)^2+1$. By the translation principle we get

$$\begin{aligned}\mathcal{L}^{-1}\left\{\frac{2s+2}{(s^2-6s+10)^2}\right\} &= 2\mathcal{L}^{-1}\left\{\frac{(s-3)+3+1}{((s-3)^2+1)^2}\right\}\\ &= 2e^{3t}\left(\mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} + 4\mathcal{L}^{-1}\left\{\frac{1}{(s^2+1)^2}\right\}\right)\\ &= 2e^{3t}\left(\frac{1}{2}t\sin t + 4(\frac{1}{2}(\sin t - t\cos t)\right)\\ &= -4te^{3t}\cos t + (t+4)e^{3t}\sin t.\end{aligned}$$