Instructions. Do each problem showing your work. Answers alone are not sufficient. Label each problem clearly, and write neatly, in a logical sequence.

Do the following problems from the text:
Page 177: 12, 24
Page 185: 16, 20, 24
Page 235: 6, 10, 16
Page 291: 2, 8, 10
Page 243: 10, 16, 20

Page 177:

12. $q(s) = s^2 - 10s + 25 = (s-5)^2$, hence the root of $q(s)$ is 5 with multiplicity 2. Thus $\mathcal{B}_q = \{e^{5t}, te^{5t}\}$

24. We complete the square to get $q(s) = ((s-4)^2 + 2^2)^3$. Thus $q(s)$ has complex roots $4 \pm 2i$ with multiplicity 3. If follows that
$\mathcal{B}_q = \{e^{4t}\cos 2t, e^{4t}\sin 2t, te^{4t}\cos 2t, te^{4t}\sin 2t, t^2e^{4t}\cos 2t, t^2e^{4t}\sin 2t\}$

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16. The roots are -8 with multiplicity 2 and $\pm 3i$ with multiplicity 3. Hence $\mathcal{B}_q = \{e^{-8t},\ te^{-8t},\ \cos 3t,\ \sin 3t,\ t\cos 3t,\ t\sin 3t,\ t^2\cos 3t,\ t^2\sin 3t\}$.

20. The roots are 1 with multiplicity 1, 2 with multiplicity 2, and and 3 with multiplicity 3. Hence $\mathcal{B}_q = \{e^t,\ e^{2t},\ te^{2t},\ e^{3t},\ te^{3t},\ t^2e^{3t}\}$.

24. $s^3 + 8 = (s+2)(s^2 - 2s + 4) = (s+2)(s^2 - 2s + 1 + 3) = (s+2)((s-1)^2 + (\sqrt{3})^2)$; hence $\mathcal{B}_q = \left\{e^{-2t},\ e^t\cos\sqrt{3}t,\ e^t\sin\sqrt{3}t\right\}$

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6. The characteristic polynomial is $q(s) = s^2 - 3s - 10 = (s-5)(s+2)$ so $\mathcal{B}_q = \{e^{5t}, e^{-2t}\}$ and the general solution takes the form $y(t) = c_1e^{5t} + c_2e^{-2t}$, $c_1, c_2 \in \mathbb{R}$

10. The characteristic polynomial is $q(s) = s^2 + 8s + 25 = (s+4)^2 + 9$ so $\mathcal{B}_q = \{e^{-4t}\cos 3t, e^{-4t}\sin 3t\}$ and the general solution takes the form $y(t) = c_1e^{-4t}\cos 3t + c_2e^{-4t}\sin 3t$, $c_1, c_2 \in \mathbb{R}$

16. The characteristic polynomial is $q(s) = s^2 + 4s + 13 = (s+2)^2 + 9$ so $\mathcal{B}_q = \{e^{-2t}\cos 3t, e^{-2t}\sin 3t\}$ and the general solution takes the form $y(t) = c_1e^{-2t}\cos 3t + c_2e^{-2t}\sin 3t$. The initial conditions imply that $c_1 = 1$ and $-2c_1 + 3c_2 = -5$. Solving gives $c_1 = 1$ and $c_2 = -1$. Thus $y = e^{-2t}\cos 3t - e^{-2t}\sin 3t$.

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2. The characteristic polynomial is $q(s) = s^3 - 6s^2 + 12s - 8 = (s-2)^3$. Thus $\mathcal{B}_q = \{e^{2t}, te^{2t}, t^2e^{2t}\}$. It follows that $y(t) = (c_1 + c_2t + c_3t^2)e^{2t}$

8. The characteristic polynomial is $q(s) = (s^2 + 9)^3$. Thus

$$\mathcal{B}_q = \left\{\cos 3t, \sin 3t, t\cos 3t, t\sin 3t, t^2\cos 3t, t^2\sin 3t\right\}.$$

It follows that $y(t) = c_1\cos 3t + c_2\sin 3t + c_3 t\cos 3t + c_4 t\sin 3t + c_5 t^2\cos 3t + c_6 t^2\sin 3t$.

10. The characteristic polynomial is $q(s) = s^3 + s^2 - s - 1 = (s-1)(s+1)^2$. Thus $\mathcal{B}_q = \{e^t, e^{-t}, te^{-t}\}$. It follows that $y(t) = c_1e^t + c_2e^{-t} + c_3te^{-t}$. Since

$$\begin{aligned} y(t) &= c_1e^t + c_2e^{-t} + c_3te^{-t} \\ y'(t) &= c_1e^t - c_2e^{-t} + c_3(1-t)e^{-t} \\ y''(t) &= c_1e^t + c_2e^{-t} + c_3(-2+t)e^{-t} \end{aligned}$$

we have

$$\begin{aligned} 1 = y(0) &= c_1 + c_2 \\ 4 = y'(0) &= c_1 - c_2 + c_3 \\ -1 = y''(0) &= c_1 + c_2 - 2c_3 \end{aligned}$$

and hence $c_1 = 2$, $c_2 = -1$, and $c_3 = 1$. Thus $y = 2e^t - e^{-t} + te^{-t}$.

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10. The characteristic polynomial is $q(s) = s^2 + 3s - 4 = (s+4)(s-1)$. Since $\mathcal{L}\{e^{2t}\} = 1/(s-2)$, we set $v(s) = s - 2$. Then $q(s)v(s) = (s+4)(s-1)(s+2)$. Since $\mathcal{B}_{qv} = \{e^{-4t}, e^t, e^{2t}\}$ and $\mathcal{B}_q = \{e^{-4t}, e^t\}$ we have $y_p = a_1e^{2t}$, a test function. Substituting y_p into the differential equation gives $6a_1e^{2t} = e^{2t}$. It follows that $a_1 = 1/6$. The general solution is $y = \frac{1}{6}e^{2t} + c_1e^t + c_2e^{-4t}$.

16. We use the principle of superposition and break this problem into two separate parts: $y'' + 4y = 1$ and $y'' + 4y = e^t$. For $y'' + 4y = 1$: The characteristic polynomial is $q(s) = s^2 + 4$. Since $\mathcal{L}\{1\} = 1/s$, we set $v(s) = s$. Then $q(s)v(s) = (s^2+4)s$. Since $\mathcal{B}_{qv} = \{\cos 2t, \sin 2t, 1\}$ and $\mathcal{B}_q = \{\cos 2t, \sin 2t\}$ we have $y_{p_1} = a_1$, a test function. Substituting y_{p_1} into the differential equation gives $4a_1 = 1$. It follows that $a_1 = 1/4$ and $y_{p_1} = 1/4$. For $y'' + 4y = e^t$: Since $\mathcal{L}\{e^t\} = 1/(s-1)$ we set $v(s) = s - 1$. Then $q(s)v(s) = (s^2+4)(s-1)$ and $\mathcal{B}_{qv} = \{\cos 2t, \sin 2t, e^t\}$. It follows that $y_{p_2} = a_1e^t$ is a test function. Substituting y_{p_2} into the differential equation gives $5a_1e^t = e^t$ from which we get $a_1 = 1/5$. Thus $y_{p_2} = (1/5)e^t$. The general solution is $y = \frac{1}{4} + \frac{1}{5}e^t + c_1\cos(2t) + c_2\sin(2t)$.

20. The characteristic polynomial is $q(s) = s^2 + 1$, an irreducible quadratic. Since $\mathcal{L}\{2\sin t\} = 2/(s^2+1)$, we set $v(s) = s^2 + 1$. Then $q(s)v(s) = (s^2+1)^2$. Since $\mathcal{B}_{qv} = \{\cos t, \sin t, t\cos t, t\sin t\}$ and $\mathcal{B}_q = \{\cos t, \sin t\}$ we have $y_p = a_1t\cos t + a_2t\sin t$, a test function. Substituting y_p into the differential equation gives $-2a_1\sin t + 2a_2\cos t = 2\sin t$. Linear independence implies $-2a_1 = 2$ and $2a_2 = 0$. Thus $a_1 = -1$ and $a_2 = 0$. The general solution is $y = -t\cos t + c_1\sin t + c_2\cos t$.