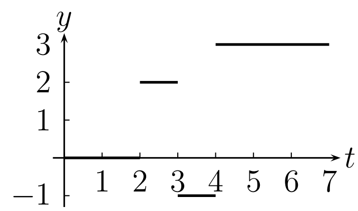
Instructions. Do each problem showing your work. Answers alone are not sufficient. Label each problem clearly, and write neatly, in a logical sequence.

Do the following problems from the text:
Page 411: 2, 4, 10, 14, 22, 28, 34, 38

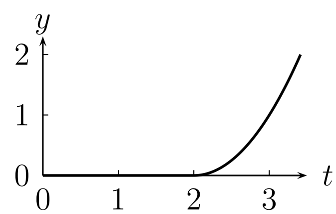
2.

$$f(t) = 2h(t-2) - 3h(t-3) + 4h(t-4) = \begin{cases} 0 & \text{if } t < 2, \\ 2 & \text{if } 2 \le t < 3, \\ -1 & \text{if } 2 \le t < 4, \\ 3 & \text{if } t \ge 4. \end{cases}$$

Thus, the graph is



4. This function is $g(t-2)h(t-2)$ where $g(t) = t^2$, so the graph of $f(t)$ is the graph of $g(t) = t^2$ translated 2 units to the right and then truncated at $t = 2$, with the graph before $t = 2$ replaced by the line $y = 0$. Thus the graph is



10. (a) $t\chi_{[2,\infty)}(t)$;

(b) $th(t-2)$;

(c) $\mathcal{L}\{th(t-2)\} = e^{-2s}\mathcal{L}\{t+2\} = e^{-2s}\left(\frac{1}{s^2} + \frac{2}{s}\right)$.

14. (a) $(t^2-4)\chi_{[4,\infty)}(t)$;

(b) $(t^2-4)h(t-4)$;

(c) $$\mathcal{L}\{(t^2-4)h(t-4)\} = e^{-4s}\mathcal{L}\{(t+4)^2-4\} = e^{-4s}\mathcal{L}\{t^2+8t+12\} = e^{-4s}\left(\frac{2}{s^3} + \frac{8}{s^2} + \frac{12}{s}\right).$$

22. (a) $t\chi_{[0,1)}(t) + (2-t)\chi_{[1,2)}(t) + \chi_{[2,\infty)}(t)$;

(b) $t - (2-2t)h(t-1) + (t-1)h(t-2)$;

(c) $\mathcal{L}\left\{t + (2-2t)h(t-1) + (t-1)h(t-2)\right\}$
$= \mathcal{L}\left\{t\right\} + e^{-s}\mathcal{L}\left\{(2-2(t+1))\right\} + e^{-2s}\mathcal{L}\left\{(t+2)-1\right\}$
$= \mathcal{L}\left\{t\right\} + e^{-s}\mathcal{L}\left\{-2t\right\} + e^{-2s}\mathcal{L}\left\{t+1\right\}$
$= \dfrac{1}{s^2} - \dfrac{2e^{-s}}{s^2} + e^{-2s}\left(\dfrac{1}{s^2} + \dfrac{1}{s}\right).$

28. $\mathcal{L}^{-1}\left\{\dfrac{e^{-3s}}{s-1}\right\} = h(t-3)\,\mathcal{L}^{-1}\left\{\dfrac{1}{s-1}\right\}\Bigg|_{t\to t-3}$
$= h(t-3)\,(e^t)\big|_{t\to t-3} = e^{t-3}h(t-3) = \begin{cases} 0 & \text{if } 0 \le t < 3, \\ e^{t-3} & \text{if } t \ge 3. \end{cases}$

34. $\mathcal{L}^{-1}\left\{\dfrac{e^{-s}}{s^2} + \dfrac{e^{-2s}}{(s-1)^3}\right\}$
$= h(t-1)\,\mathcal{L}^{-1}\left\{\dfrac{1}{s^2}\right\}\Bigg|_{t\to t-1} + h(t-2)\,\mathcal{L}^{-1}\left\{\dfrac{1}{(s-1)^3}\right\}\Bigg|_{t\to t-2}$
$= h(t-1)\,(t)\big|_{t\to t-1} + h(t-2)\,(\tfrac{1}{2}t^2e^t)\big|_{t\to t-2}$
$= (t-1)h(t-1) + \tfrac{1}{2}(t-2)^2e^{t-2}h(t-2)$
$= \begin{cases} 0 & \text{if } 0 \le t < 1, \\ t-1 & \text{if } 1 \le t < 2, \\ t-1+\tfrac{1}{2}(t-2)^2e^{t-2} & \text{if } t \ge 2. \end{cases}$

38. $\mathcal{L}^{-1}\left\{\dfrac{e^{-2s}+e^{-3s}}{s^2-3s+2}\right\} = h(t-2)\,\mathcal{L}^{-1}\left\{\dfrac{1}{s^2-3s+2}\right\}\Bigg|_{t\to t-2} + h(t-3)\,\mathcal{L}^{-1}\left\{\dfrac{1}{s^2-3s+2}\right\}\Bigg|_{t\to t-3}$
$= h(t-2)\,\mathcal{L}^{-1}\left\{\dfrac{1}{s-2} - \dfrac{1}{s-1}\right\}\Bigg|_{t\to t-2}$
$+ h(t-3)\,\mathcal{L}^{-1}\left\{\dfrac{1}{s-2} - \dfrac{1}{s-1}\right\}\Bigg|_{t\to t-3}$
$= h(t-2)\,(e^{2t}-e^t)\big|_{t\to t-2} + h(t-3)\,(e^{2t}-e^t)\big|_{t\to t-3}$
$= h(t-2)\left(e^{2(t-2)} - e^{t-2}\right) + h(t-3)\left(e^{2(t-3)} - e^{t-3}\right)$

Page 425: 2, 12

2. We write the forcing function as $f(t) = -5\chi_{[0,1)} + 5\chi_{[1,\infty)} = -5 + 10h(t-1)$. Applying the Laplace transform, partial fractions, and simplifying gives

$$\begin{aligned} Y(s) &= \frac{1}{s+5} - \frac{5}{s(s+5)} + \frac{10}{s(s+5)}e^{-s} \\ &= \frac{2}{s+5} - \frac{1}{s} + \left(\frac{2}{s} - \frac{2}{s+5}\right)e^{-s}. \end{aligned}$$

Laplace inversion now gives

$$\begin{aligned} y &= 2e^{-5t} - 1 + \left(2 - 2e^{-5(t-1)}\right) h(t-1) \\ &= \begin{cases} 2e^{-5t} - 1 & \text{if } 0 \le t < 1 \\ 2e^{-5t} + 1 - 2e^{-5(t-1)} & \text{if } 1 \le t < \infty \end{cases}. \end{aligned}$$

12. Write the forcing function as $f(t) = e^{-t}\chi_{[0,4)} = e^{-t} - e^{-t}h(t-4)$ and then $\mathcal{L}\{f(t)\} = \frac{1}{s+1} - e^{-4s}\mathcal{L}\left\{e^{-(t+4)}\right\} = \frac{1}{s+1} - \frac{1}{s+1}e^{-4}e^{-4s}$. Apply the Laplace transform, partial fractions, and simplify to get

$$Y(s) = \frac{1}{(s+1)^3} - \frac{1}{(s+1)^3}e^{-4}e^{-4s}.$$

Laplace inversion then gives

$$\begin{aligned} y &= \frac{t^2}{2}e^{-t} - \frac{(t-4)^2}{2}e^{-(t-4)}e^{-4}h(t-4) \\ &= \frac{t^2}{2}e^{-t} - \frac{(t-4)^2}{2}e^{-t}h(t-4) \\ &= \begin{cases} \frac{t^2}{2}e^{-t} & \text{if } 0 \le t < 4 \\ (4t-8)e^{-t} & \text{if } 4 \le t < 0 \end{cases}. \end{aligned}$$

Page 437: 6, 8

6. Apply the Laplace transform, take partial fractions, and simplify to get

$$\begin{aligned} Y(s) &= \frac{e^{-s}}{s^2-1} - \frac{e^{-2s}}{s^2-1} \\ &= \frac{1}{2}\left(\frac{1}{s-1} - \frac{1}{s+1}\right)e^{-s} - \frac{1}{2}\left(\frac{1}{s-1} - \frac{1}{s+1}\right)e^{-2s}. \end{aligned}$$

Laplace inversion now gives

$$\begin{aligned} y &= \frac{1}{2}\left(e^{t-1} - e^{-(t-1)}\right)h(t-1) - \frac{1}{2}\left(e^{t-2} - e^{-(t-2)}\right)h(t-2) \\ &= \frac{1}{2}\begin{cases} 0 & \text{if } 0 \le t < 1 \\ e^{t-1} - e^{-(t-1)} & \text{if } 1 \le t < 2 \\ e^{t-1} - e^{-(t-1)} - e^{t-2} + e^{-(t-2)} & \text{if } 2 \le t < \infty \end{cases}. \end{aligned}$$

8. Apply the Laplace transform to get

$$Y(s) = \frac{s}{s^2+4} + \frac{1}{s^2+4}e^{-\pi s} - \frac{1}{s^2+4}e^{-2\pi s}.$$

Laplace inversion gives

$$\begin{aligned}
y &= \cos 2t + \frac{1}{2}\sin(2(t-\pi))h(t-\pi) - \frac{1}{2}\sin(2(t-2\pi))h(t-2\pi) \\
&= \cos 2t + \frac{1}{2}(\sin 2t)h(t-\pi) - \frac{1}{2}(\sin 2t)h(t-2\pi) \\
&= \begin{cases} \cos 2t & \text{if } 0 \le t < \pi \\ \cos 2t + \frac{1}{2}\sin 2t & \text{if } \pi \le t < 2\pi \\ \cos 2t & \text{if } 2\pi \le t < \infty \end{cases} .
\end{aligned}$$