

Math 2070 Section 1
Review Exercises for Exam I

The syllabus for Exam I is Chapter 8 (Sections 1 – 4), Sections 1.1, 1.3, and 1.4.

You should bring your own paper for the exam, since space will (in general) not be provided on the exam paper for answers. You will need to show all relevant work in order to receive partial credit.

1. Let $A = \begin{bmatrix} 8 & x \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$. Determine all values of x for which $AB = BA$.
2. Let A be a 3×1 matrix, B a 2×3 matrix, and C a 2×1 matrix. Which of the following matrix products is well defined and the result is a 1×2 matrix?
 - (a) AB
 - (b) $B^T C$
 - (c) $C^T B A$
 - (d) $B A C^T$
 - (e) $(B A)^T$
3. Find all solutions of the following system of linear equations. Be sure to show all your steps!

$$\begin{array}{rccccrcr} x_1 & + & 2x_2 & + & 3x_3 & + & x_4 & = & 8 \\ x_1 & + & 3x_2 & & & + & x_4 & = & 7 \\ x_1 & & & + & 2x_3 & + & x_4 & = & 3 \end{array}$$

4. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$.

- (a) Compute A^{-1} .
- (b) Using your answer to part (a), solve the system of equations

$$\begin{array}{rccccrcr} x_1 & - & x_2 & & & = & 1 \\ 2x_1 & & & + & x_3 & = & 1 \\ & & x_2 & - & x_3 & = & 1 \end{array}$$

5. Find conditions that b_1 , b_2 , and b_3 must satisfy for the following system to be consistent:

$$\begin{array}{rccccrcr} x_1 & + & 2x_2 & - & 3x_3 & = & b_1 \\ 2x_1 & + & 3x_2 & + & 3x_3 & = & b_2 \\ 5x_1 & + & 9x_2 & - & 6x_3 & = & b_3 \end{array}$$

6. (a) Compute the determinant of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & k & -1 \\ 1 & k^2 & 1 \end{bmatrix}.$$

- (b) Find all the values of k for which A is **not** invertible.

7. Determine if each of the following equations is separable (S), linear (L), or neither (N). List all types that apply for each equation. Some algebraic manipulation may be needed to put the equation in the standard form for determining each type. Do **not** solve the equations!

(a) $t^2y' = 1 - 2ty$

(b) $yy' = 3 - 2y$

(c) $y' = y^2 - y$

(d) $y' = y^2 - t$

(e) $ty' = y - 2ty$

(f) $t + y' = y - 2ty$

8. Find the general solution of each of the following differential equations. You **must** show your work.

(a) $y' + 2y = 0$

(b) $y' + 2y = 3e^t$

(c) $y' - y = e^{3t}$

(d) $y' + 2ty = t$

(e) $y' + \frac{3}{t}y = \frac{\sin t}{t^3}$

(f) $y' = \frac{t^2}{y}$

(g) $y' = y^2 + 4$

9. Solve each of the following initial value problems. You **must** show your work.

(a) $y' = 2y + 5e^{2t}$, $y(0) = -1$.

(b) $y' = y^2t^3$, $y(1) = -1$.

(c) $y' + 3y = 4e^{-3t} \sin 2t$, $y(0) = -1$.

(d) $y' + \frac{3}{t}y = 7t^3$, $y(1) = -1$.

10. Newton's law of cooling states that the rate at which a body cools (or heats up) is proportional to the difference between the temperature of the body and the temperature of the surrounding medium. A turkey which is initially at room temperature (70° F) is placed in a 350° F oven at time $t = 0$. Write an initial value problem which is satisfied by the temperature $T(t)$ of the turkey at time t .
11. A tank contains 300 gal of brine made by dissolving 50 lb of salt in water. A salt solution containing 2 lb per gallon of water runs into the tank at the rate of 3 gal/min, and the mixture, which is kept uniform by stirring, runs out at the same rate.
 - (a) Find the amount $y(t)$ of salt in the tank at any time t .
 - (b) What is $\lim_{t \rightarrow \infty} y(t)$? Does your answer make sense?