

Instructions. Answer each of the questions on your own paper. Be sure to show your work so that partial credit can be adequately assessed. *Credit will not be given for answers (even correct ones) without supporting work.* Put your name on each page of your paper.

1. [16 Points] Find all solutions of the following system of linear equations. Be sure to show all your steps!

$$\begin{aligned}x_1 + 2x_2 - 2x_3 &= -8 \\3x_1 + x_2 + 9x_3 &= 1 \\x_1 + 4x_3 &= 2\end{aligned}$$

2. [16 Points] Let $A = \begin{bmatrix} x-2 & 4 & 3 \\ 1 & x+1 & -2 \\ 0 & 0 & x-4 \end{bmatrix}$.

- (a) Compute $\det A$.
- (b) For which values of x is the matrix *not* invertible.
3. [16 Points] Solve the initial value problem: $y' - 4y = 6e^{4t} + 3$, $y(0) = -2$.
4. [16 Points] Solve the initial value problem: $ty' - 3y = t^5$ $y(1) = 3$.
5. [16 Points] Solve the initial value problem: $y' = y^2(1+t)$, $y(0) = -2$.
6. [20 Points] A tank initially contains 300 gallons of a salt solution made by dissolving 30 pounds of salt in water. A solution containing 0.6 pounds of salt per gallon enters the tank at a rate of 5 gallons per minute. A drain is opened at the bottom of the tank through which the well stirred solution leaves the tank at the same rate of 5 gallons per minute. Let $y(t)$ denote the amount of salt (in pounds) which is in the tank at time t .
- (a) What is $y(0)$?
- (b) Write the differential equation that $y(t)$ must satisfy.
- (c) Solve the differential equation to find $y(t)$.
- (d) How much salt is in the tank after 1/2 hour?