

Math 2070 Sec. 1 Exam V April 30, 1996

Instructions. Answer each of the questions on your own paper. Be sure to show your work so that partial credit can be adequately assessed. The value of each problem is shown in parentheses to the left of the problem.

Please *print* your name and student number in the space provided below, and turn in this sheet with your solution papers.

Name:

Student Number:

- (35) 1. Let $f(x)$ be the periodic function of period 4 defined on the interval $[-2, 2)$ by

$$f(x) = \begin{cases} 1+x & \text{if } -2 \leq x < 0 \\ 1-x & \text{if } 0 \leq x < 2 \end{cases}$$

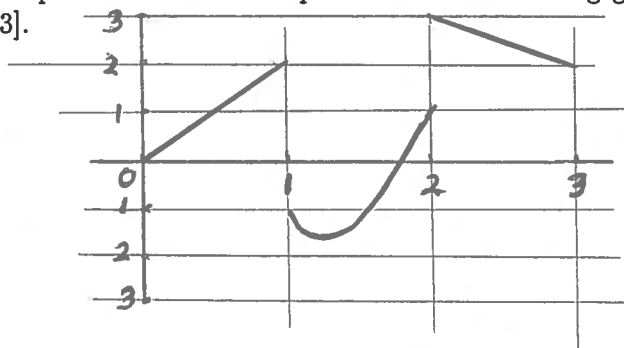
- (a) Sketch the graph of $f(x)$ on the interval $[-6, 6]$. Is $f(x)$ continuous on this interval?
 (b) Compute the Fourier series of $f(x)$.

The following integrals may be of use:

$$\int x \sin ax \, dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax + C$$

$$\int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax + C$$

- (15) 2. Let $f(x)$ be a periodic function of period 3. The following graph is the graph of $f(x)$ on the interval $[0, 3]$.



- (a) Sketch the graph of $f(x)$ on the interval $[-3, 6]$.
 (b) Let $g(x)$ be the sum of the Fourier series of $f(x)$. Determine the following values of $g(x)$: $g(0)$, $g(2)$, $g(4)$, $g(-1)$, and $g(1/2)$.
- (25) 3. Let $f(x) = \begin{cases} \sin x & \text{if } 0 < x \leq \pi \\ 0 & \text{if } -\pi < x \leq 0 \end{cases}$ and let $g(x) = x$ for $-\pi < x < \pi$. Both functions are extended to be periodic of period 2π . Fourier series expansions of these functions are as follows:

$$f(x) \sim \frac{1}{\pi} + \frac{1}{2} \sin x - \frac{2}{\pi} \left(\sum_{n=\text{even}} \frac{\cos nx}{n^2 - 1} \right)$$

$$g(x) \sim 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx$$

Using these series expansions, answer the following questions:

- (a) Compute the Fourier series of the function $g_1(x) = x^2$ for $-\pi \leq x \leq \pi$.
(b) Compute the Fourier series of the function

$$f_1(x) = \begin{cases} \cos x & \text{if } 0 < x < \pi \\ 0 & \text{if } -\pi < x < 0 \end{cases}$$

- (c) Can the Fourier series of $f''(x)$ be computed by term by term differentiation (twice) of the Fourier series of $f(x)$? Explain.
- (25) 4. Find all periodic solutions (of period 2) to the differential equation

$$y'' + 10y = \sum_{n=1}^{\infty} \left(\frac{\cos n\pi x}{n^2} + \frac{\sin n\pi x}{n} \right).$$